



# Column Generation for IMRT Cancer Therapy Optimization with Implementable Segments

by Felisa Preciado-Walters, PhD, Kimberly-Clark Corp

Ronald L. Rardin, PhD, Purdue University

Mark P. Langer, MD and Van Thai, Indiana University

School of Medicine

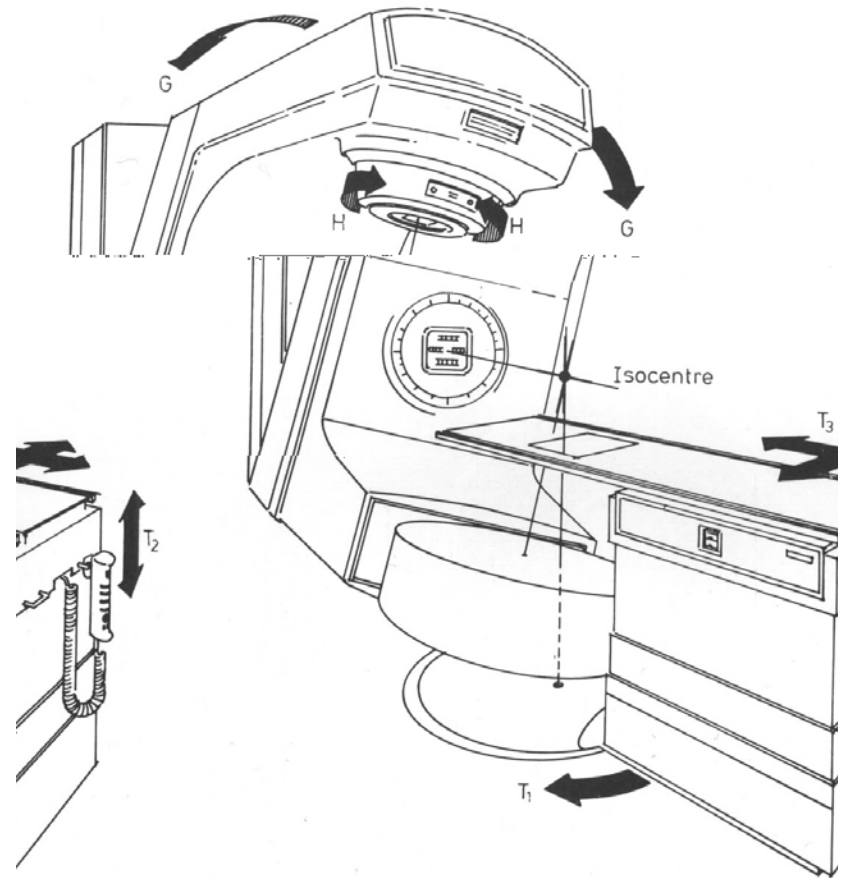
(presented by Delal Dink, Purdue University)

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# Radiation Therapy Opt



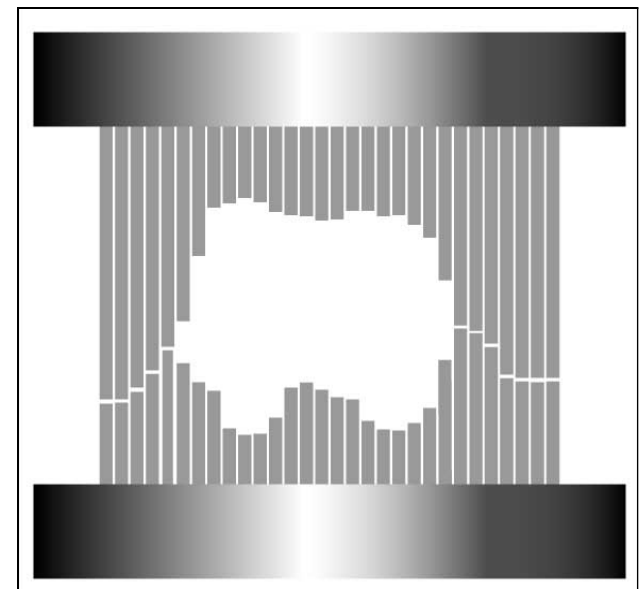
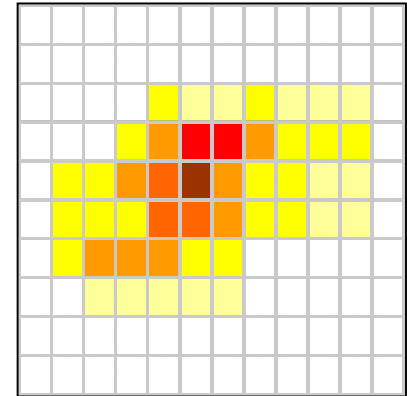
- 1.2M/yr new cases of cancer in U.S. alone
  - Times 10 globally
- Half undergo radiation therapy
- Some treated with implants, but most with **external beam**



# IMRT Refinement



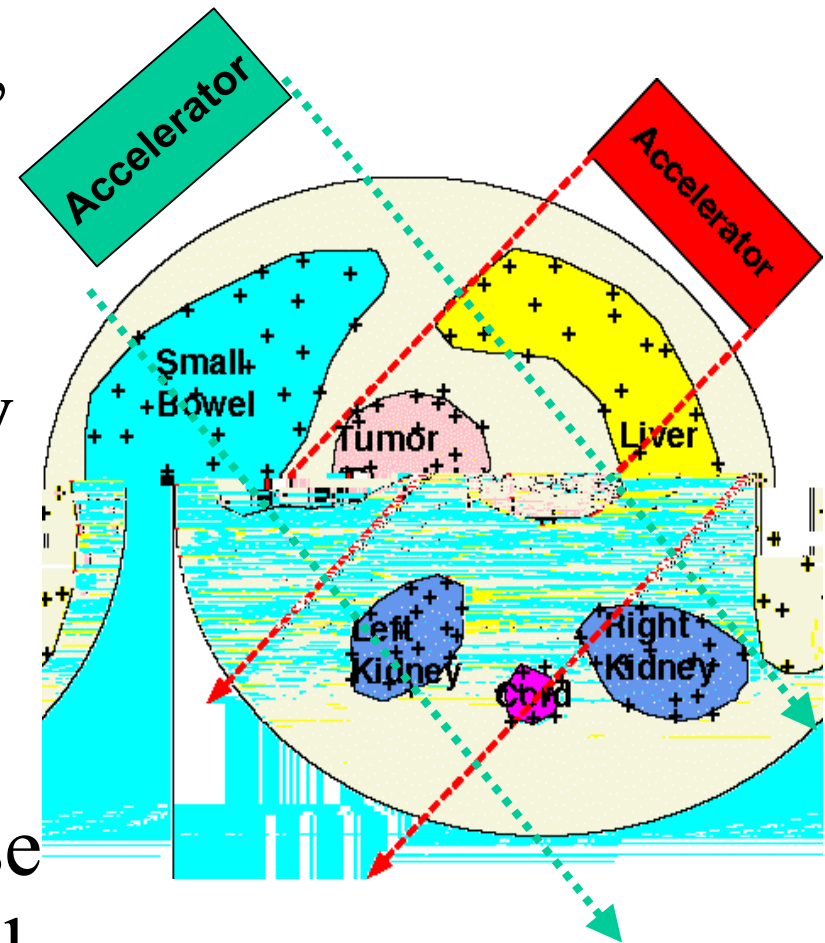
- Beam is big: 10 cm square
- Conformal therapy covers whole image of tumor
- Latest IMRT can deliver shaped array of 3mm beamlets
- Implemented via a Multi-Leaf Collimator (MLC) creating a time-varying opening (leaves can be vertical or horizontal)



# Therapy Planning Task



- Choose beam angles, shapes & intensities
- Max tumor dose
- Limit dose to healthy tissues
- Multiple beams spread healthy dose
- 1% higher tumor dose adds 1.5% to survival

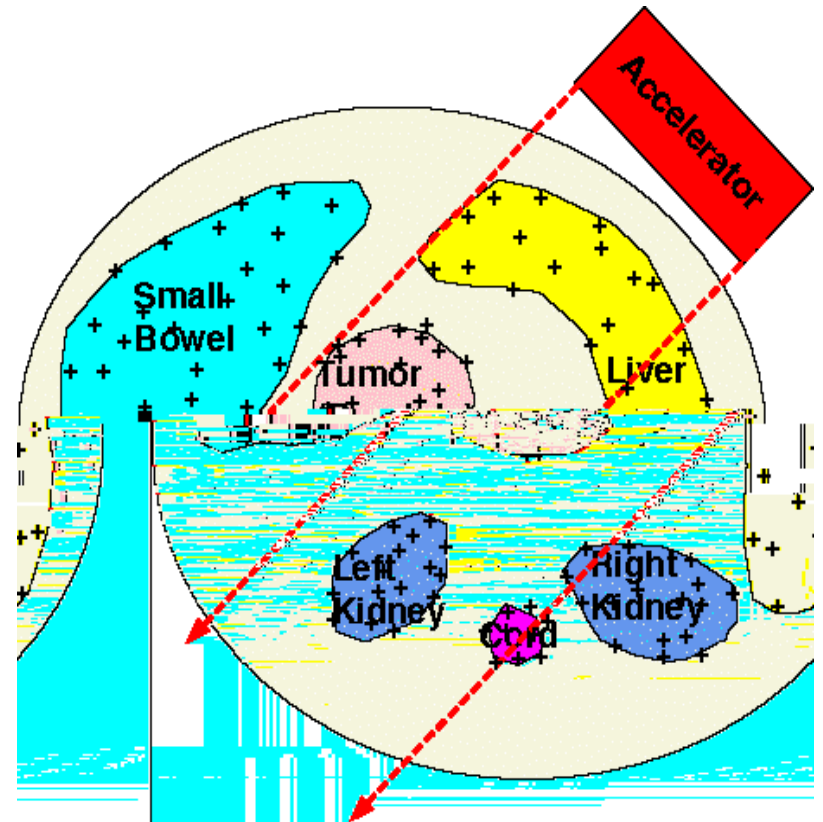


# Modeling Simplifications



- Shapes: Deal with shapes by **discrete points**
  - Hundreds per tissue
- **Linearity**: Dose at any point  $i$  linear in beam(let) intensity  $j$

$$\sum a_{ij} x_j$$



# Math Program Model



- **Objective Function:**  
max minimum dose at  
any tumor point
- **Homogeneity:** highest  
dose tumor point no  
more than fixed  
multiple of lowest

<i>Tissue</i>	<i>No. of points</i>
<i>Tumor</i>	2385
<i>Bladder</i>	751
<i>Rectum</i>	871
<i>Femoral Head 1</i>	714
<i>Femoral Head 2</i>	720
<i>Sigmoid</i>	1154
<i>Skin</i>	1811

# Math Program Model



- Simple dose limits: radiation at any healthy tissue point cannot exceed a specified limit  $b$
- Restrict number of beam angles

<i>Tissue</i>	<i>No. of points</i>	<i>b (cGy)</i>
<i>Tumor</i>	2385	-
<i>Bladder</i>	751	10000
<i>Rectum</i>	871	10000
<i>Femoral Head 1</i>	714	7200
<i>Femoral Head 2</i>	720	7200
<i>Sigmoid</i>	1154	8500
<i>Skin</i>	1811	15000

# Math Program Model



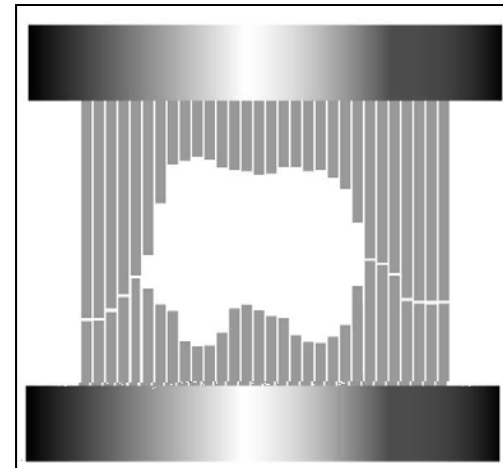
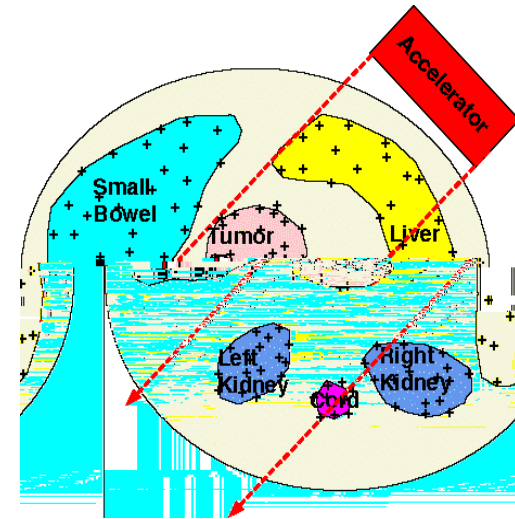
- **Dose Volume:** constraints restricting the load to  $d < b$  on a fraction of a tissue

<i>Tissue</i>	<i>No. of points</i>	<i>b (cGy)</i>	<i>% under DV rest.</i>	<i>d (cGy)</i>
<i>Tumor</i>	2385	-	-	-
<i>Bladder</i>	751	10000	80	8000
<i>Rectum</i>	871	10000	80	7500
<i>Femoral Head 1</i>	714	7200	60	5000
<i>Femoral Head 2</i>	720	7200	60	5000
<i>Sigmoid</i>	1154	8500	90	7500
<i>Skin</i>	1811	15000	0	-

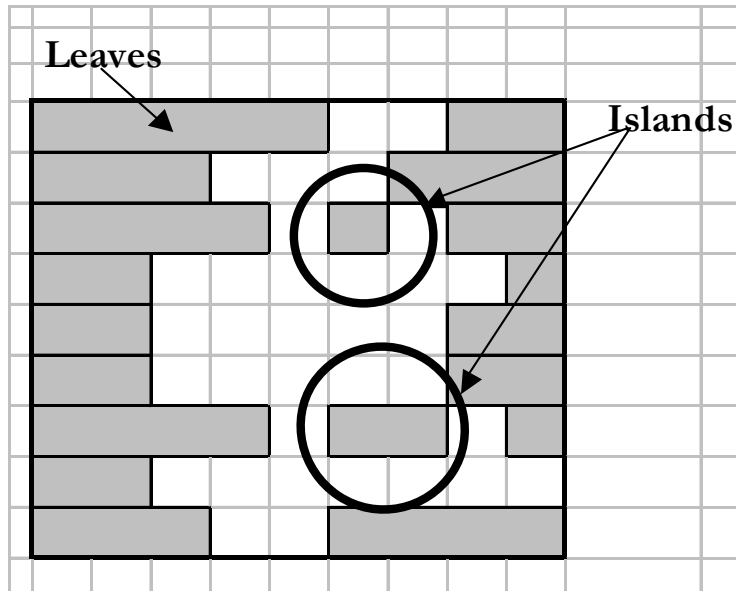
# Mixed-Integer Approach



- Model **dose-volume** and **beam-count** constraints with **binary variables**
- **Column-generate** to find good **IMRT segments** (open/close)
- **Round** LP relaxations to produce MIP solns

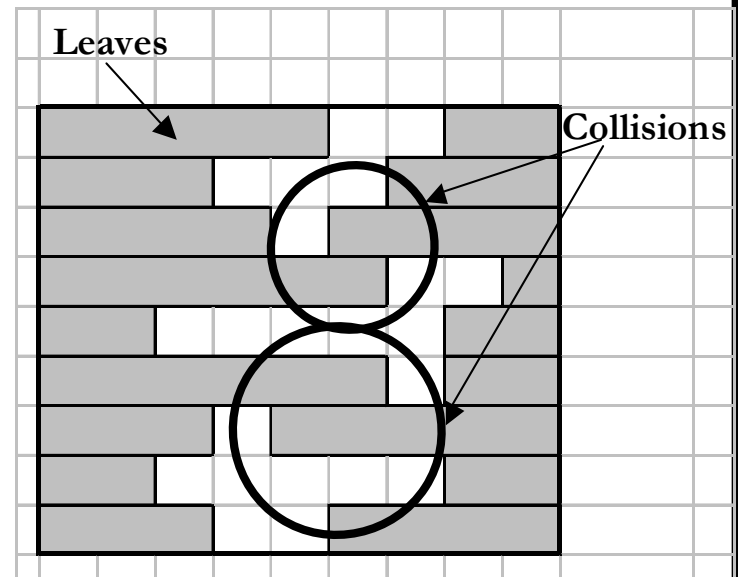


# MLC Segment Limitations



- **Collision** risk precludes leaf overlap in adjacent rows

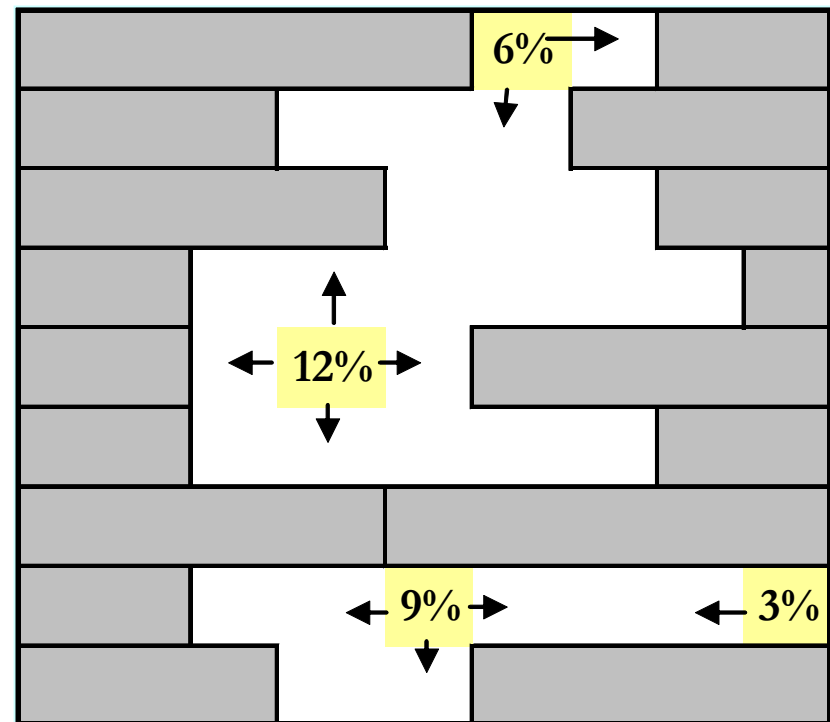
- **Islands** in rows are not implementable with MLCs



# MLC Boundary Corrections



- **Tongue and Grove** design of leaves also introduces boundary effects
- We adjust effective beamlet intensities +3% for every open neighbor



# Implementable Segment Column Generation



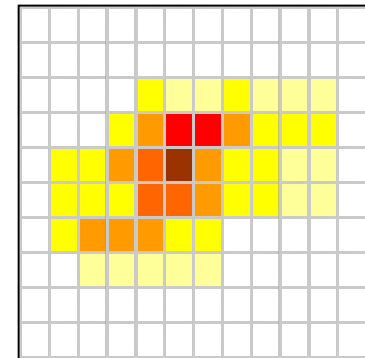
- Employ classic approach of building column via dual variable values on constraints
  - Sum to a **potential** =  $cbar$
- Select positives to open, enforcing rules for Island and Collision ( $P$ -time)
- Apply 3% adjustment and add to LP relaxation

-4	5	-1	-5	-8	-2	-3	0	0	0
0	-4	-1	8	7	0	-1	-9	9	-8
0	-2	6	-1	4	-5	-8	-9	-9	-7
-2	-5	-3	2	3	-7	-5	-7	-8	-5
-5	-3	-7	-4	2	-3	-8	-5	-4	-2
-6	-7	-5	-5	-3	6	9	11	2	0
-7	8	-8	-7	2	10	3	6	-2	-1
-8	-8	-9	5	-9	-4	0	9	-3	-1
-6	-9	-4	-3	-1	-5	-8	-2	-3	0
-5	-8	-8	-7	-4	-6	-2	5	3	0

# An Alternative to Two-Stage Approach



- Our MIP column generation approach produces segments and intensities that can be implemented directly
- Most current methods use **two stages**
  - First, optimize treating beamlets as independent
  - Second, derive a segment plan that approximately delivers optimal beamlet intensities
    - Discretize intensities to 10-15 values
    - Apply segment a generation heuristic
    - Adjust the results for boundary effects



# Comparison of Run Times



- In most cases complete (rounded) MIP runs take less CPU than independent beamlets LP alone
- Typically runs more efficiently when MLC segment rules limit column generation

Target Site	Independent Beamlets	Colgen without MLC Segment Rules		Colgen with MLC Segment Rules	
	CPU (mins)	Number Segments	CPU (mins)	Number Segments	CPU (mins)
prostate	452.1	233	29.4	202	19.4
prostate	609.1	137	23.2	146	13.8
prostate	168.8	253	11.3	217	6.8
lung	27.4	822	61.9	560	31.0
naso-pharynx	4.1	318	4.0	256	3.5
thorax	37.5	207	3.5	243	5.9

# Delivery Gains (Prostate Case)



- Can measure effects by post-evaluation with many sample pts
- Results (1 case)
  - MIP Direct avoids dangerous infeasibilities
  - MIP Direct requires only half the exposure time
  - MIP Direct uses more segments

	MIP Direct	Two Stage
Min Tumor Dose	99.6 Gy	98.4 Gy
Bladder Limit Violation (over 100Gy)	0.23 Gy	8.59 Gy
Rectum Limit Violation (over 100Gy)	0.15 Gy	8.27 Gy
Skin Limit Violation (over 150Gy)	0.00 Gy	8.67 Gy
Tumor Dose Homogeneity (vs 90%)	0.25%	10.33%
Avg Segs per Angle (9 fixed)	9.0	5.8
Total Fluence	19,723 MU	40,454 MU

# Summary



- Column generation offers a flexible approach to dealing with the large numbers of beamlets in IMRT
  - Including permitting enforcement of MLC rules
- Preliminary tests show implied direct implementation often uses less computation than the common two-stage approach and avoids potentially dangerous infeasibilities