Calculus II
Math 1312-2 - Fall 06 Name:____________________________

Review Problems for Final Exam

Go over all previous review sheets, exams, and homework. Some questions will overlap with this review. The final exam will be comprehensive and will cover 7.2-7.8, 8.1-8.6, 10.1-10.8, 10.10, 11.1-11.4, 11.6

1. Use the formula below from the table of integrals,

$$\boxed{\int u^2\sqrt{u^2+a^2}du = \frac{u}{8}(2u^2+a^2)\sqrt{u^2+a^2} - \frac{a^4}{8}\ln\left|u+\sqrt{u^2+a^2}\right| + C}$$

in order to find $\int x^2\sqrt{4x^2+9}dx$

2. Determine the partial fraction decomposition of the following rational functions.

(a) $\dfrac{1}{x^2-3x-4}$ (b) $\dfrac{x-1}{x^2+x-6}$

3. Evaluate $\int_0^{\pi} e^{-t}\cos t dt$

4. Evaluate the integrals in the following problems:

(a) $\int \tan^5 x \sec^3 x dx$ (b) $\int \cos^3 t \sin^4 t dt$

(c) $\int \sqrt{9x^2+16}dx$

(d) $\int x^5\sqrt{2-x^3}dx$

(e) $\int \frac{\sqrt{x}}{1+x}dx$

(f) $\int x\ln x dx$

(g) $\int x^2e^{-x}dx$

(h) $\int \sin^3 x\cos^5 x dx$

(i) $\int \sqrt{9-x^2}dx$

(j) $\int \frac{1}{x\sqrt{4+x^2}}dx$

(k) $\int \frac{1}{x\sqrt{6x-x^2}}dx$

(l) $\int \frac{1}{(x-1)(x-4)}dx$

5. Determine whether or not the improper integrals converge. Evaluate those that do converge.

   (a) $\int_0^2 \frac{x}{x^2-1}\,dx$

   (b) $\int_1^\infty \frac{1}{x^2}\,dx$

6. Find a solution to $\frac{dy}{dx} = \sqrt{y}\sin x$, $y(0) = 4$

7. A tank shaped like a vertical cylinder initially contains water to a depth of 16ft. A bottom plug is pulled at time $t = 0$ (t in min). After 10min the depth has dropped to 9ft. how long will it take all the water to drain from this tank?

8. The shape of a water tank is obtained by revolving the curve $y = x^2$ around the y-axis. A plug in the bottom is removed at noon when the water level is 16ft. Two hours later the level is at 1ft. When will the tank be empty?

9. Solve the following differential equations:

   (a) $\dfrac{dy}{dx} = e^x + y$

   (b) $x\dfrac{dy}{dx} + 3y = 3x^{-3/2}$

   (c) $(x^2 - 1)\dfrac{dy}{dx} + (x - 1)y = 1$

10. A tank of capacity 1000L contains 500L of a solution containing 40kg of salt dissolved. Water containing 1kg/L is pumped in the tank at a rate of 10L/min and the mixed solution is then pumped out at 5L/min. How much salt is in the tank when it is full?

11. A 200 gallon tank initially containing 10 gallons of water with 1lb of salt dissolved is being filled with brine at the rate of 4gal/min containing 2lb/gal of salt. As the solution is being mixed, 3gal/min is pumped out. When full, how much salt will be in the tank?

12. Suppose the a population (in millions) of bacteria grows according to the following logistic equation

$$\frac{dP}{dt} = 0.5P - 0.01P^2$$

After a long period of time, what do you expect the population to be?

13. In each of the problems bellow find the general solution and particular solution (when applicable) of the constant coefficient homogeneous second order linear equation

(a) $y'' + 2y' - 15y = 0$

(b) $y'' + 3y' - 18y = 0,\ y(0) = 1,\ y'(0) = 1$

(c) $y'' + 6y' + 9y = 0,\ y(0) = 3,\ y'(0) = 6$

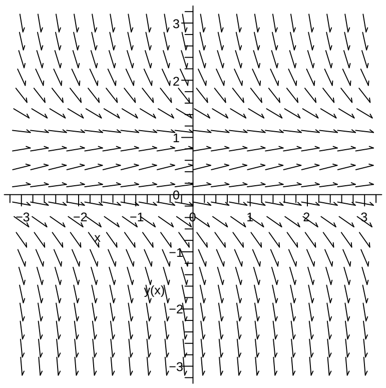
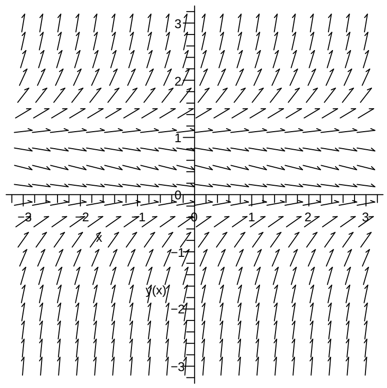
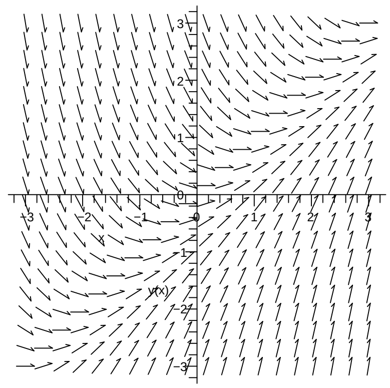
14. Determine if the ***sequence*** $a_n = \left(\frac{2}{3}\right)^n$ is convergent. If it is, determine its limit.

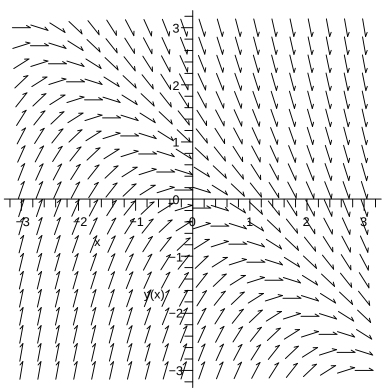
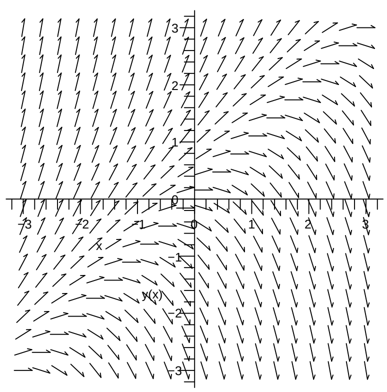
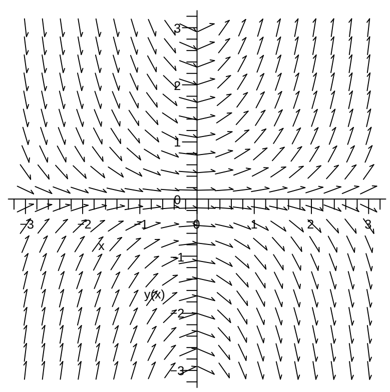
15. Write the formal definition of a convergent series, that is, what does $\sum_{n=1}^{\infty} a_n = S$ means?

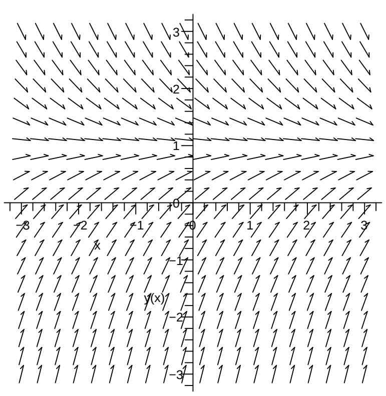
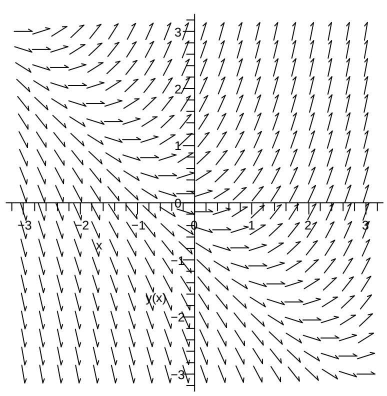
16. Match the following directional fields with the following ordinary Differential Equation (write the corresponding letter near each directional field).

(a) $\dfrac{dy}{dx} = x + y$ (b) $\dfrac{dy}{dx} = x - y$ (c) $\dfrac{dy}{dx} = y - x$ (d) $\dfrac{dy}{dx} = -y - x$

(e) $\dfrac{dy}{dx} = yx$ (f) $\dfrac{dy}{dx} = y - y^2$ (g) $\dfrac{dy}{dx} = y^2 - y$ (h) $\dfrac{dy}{dx} = 1 - y$

17. Determine if the series converges or diverges. Justify your answer by stating which test you used and providing full details of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n!}$

18. Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$ Converges? If so compute it.

19. How many terms do we need to add to approximate the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ to 4 decimal places?

20. Approximate within 3 decimal places the following definite integral

$$\int_0^1 e^{-x^2}\,dx$$

21. Determine whether the series below converges absolutely, conditionally, or diverges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2-1}$

22. Determine the radius of convergence of the following power series and indicate the interval of values where the series is convergent.

(a) $\sum_{n=1}^{\infty} \frac{n!x^n}{10^n}$

(b) $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n2^n}$

(c) $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n+1}$

23. Find the Taylor series of $f(x) = \ln(x+1)$ at $a = 0$.

24. Use the well-known Taylor series to find a power series representation of the functions below.

(a) $f(x) = x^2 e^{2x}$

(b) $f(x) = \dfrac{x}{1-x^2}$

(c) $f(x) = \dfrac{\ln(x+1)}{x}$

25. Compute the following:

(a) $\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix}$

(b) $\begin{vmatrix} 2 & 3 & 4 \\ -2 & 1 & -5 \\ 1 & 6 & 7 \end{vmatrix}$

26. Given

$A = \begin{pmatrix} 1 & 1 & 2 \\ -2 & 1 & -3 \\ 3 & -1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 5 & -1 \\ -1 & 2 & 3 \end{pmatrix}$

(a) Find $A + B$

(b) Find AB

(c) Find BA

27. Given

$A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 & 2 \\ 0 & 2 & -1 \end{pmatrix}$, and $C = \begin{pmatrix} -2 & 1 \\ 4 & 0 \\ -3 & 1 \end{pmatrix}$. Calculate the product of each two of the matrices (six in total) whichever is defined. If not possible, explain why?

28. Given $\overrightarrow{a} = \langle 1, 2, -1 \rangle$, $\overrightarrow{b} = \langle 2, 3, -2 \rangle$, and $\overrightarrow{c} = \langle -1, 1, 0 \rangle$, find:

(a) $|2\overrightarrow{a} - 3\overrightarrow{b} + 4\overrightarrow{c}|$

(b) $|\overrightarrow{a} \times \overrightarrow{b}|$

(c) $\overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c})$

(d) Equation of the line passing through $(1, 2, 1)$ in the direction of $\overrightarrow{b}$.

(e) Equation of the plane through $(0, 1, 1)$ containing the vectors $\overrightarrow{a}$ and $\overrightarrow{c}$.