

“Some” Answers of Review Problems for Exam 2

1.  $k < 0$ .
2. (a)  $u(x, t)$  is the temperature of a thin laterally insulated rod of length  $\pi$  with boundaries at a prescribed temperature of 0 and initial temperature of  $11 \sin(3x)$   
(b)  $u(x, t) = 11 \sin(3x)e^{-36t}$   
(c)  $u(x) = 0$
3. (a)  $u(x, t)$  is the temperature of a thin laterally insulated rod of length 1 with boundaries at a prescribed temperature of 0 and 2 and initial temperature of  $\sin(\pi x)$   
(b)  $u(x, t) = \sin(\pi x)e^{-16\pi^2 t} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n\pi} \sin(n\pi x) e^{-16n^2\pi^2 t}$   
(c)  $u(x) = 2x$
4. (a)  $u(x, t)$  is the temperature of a thin laterally insulated rod of length  $\pi$  with insulated boundaries initial temperature of  $3 + \cos(4x)$   
(b)  $u(x, t) = 3 + \cos(4x)e^{-100\pi^2 t}$   
(c)  $u(x) = 3$
5. (a)  $u(x, t)$  is the temperature of a thin laterally insulated rod of length 1 with boundaries at a prescribed temperature of 0 and 2 and initial temperature of  $\sin(\pi x)$   
(b)  $u(x, t) = \sin(\pi x)e^{-16\pi^2 t} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n\pi} \sin(n\pi x) e^{-16n^2\pi^2 t}$   
(c)  $u(x) = 2x$
6.  $u(x, t)$  is the temperature of a thin laterally insulated rod of length 1 with one insulated boundary and the other at a prescribed temperature of 0 with initial condition given by  $f(x)$ .

The solution is:

$$u(x, t) = \sum_{n=1}^{\infty} d_n \cos\left(n - \frac{1}{2}\right) \pi x e^{[5(n-\frac{1}{2})\pi]^2 t},$$

where  $d_n$  is given by:

$$d_n = \frac{1}{\int_0^1 \cos^2\left(n - \frac{1}{2}\right) \pi x dx} \int_0^1 f(x) \cos\left(n - \frac{1}{2}\right) \pi x dx$$

7. (a) formulas  
(b) formulas

- (c) formulas
- 8. Separate into 2 problems and use formulas from book.
- 9. Definition
- 10. (a) Yes  
(b) No
- 11.  $u_r(\cos \theta + \sin \theta) + u_\theta \left( \frac{\cos \theta - \sin \theta}{r} \right)$
- 12. Definition
- 13. Definition
- 14. discuss the asymptotic behavior as  $x$  goes to 0.
- 15. see book.