

**Problem of the Week**

**08/27/2009 to 09/04/2009**

Consider the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, ... What is the 2009<sup>th</sup> term of this sequence?

**Solution:** The 2009<sup>th</sup> term of this sequence is 63.

We see that there is 1 term before the 2's begin, 3 terms before the 3's begin, 6 terms before the 4's begin, and 10 terms before the 5's begin, and one may notice these as numbers down a diagonal of Pascal's Triangle, that is

$$1 = \binom{2}{2}, \quad 3 = \binom{3}{2}, \quad 6 = \binom{4}{2}, \quad \text{and } 10 = \binom{5}{2}.$$

So, it appears that the number  $n$  first appears in the sequence in the  $\binom{n}{2} + 1^{\text{st}}$  place. Thus, we need

$$\binom{n}{2} < 2009 < \binom{n+1}{2},$$

that is,

$$\frac{n(n-1)}{2} < 2009 < \frac{(n+1)n}{2}.$$

This yields the following inequality

$$n^2 - n < 4018 < n^2 + n.$$

Now  $63 < \sqrt{4018} < 64$ , and by checking, we see that 63 satisfies our inequality.

**Solutions for this problem were submitted by Brian Fitzpatrick, Matt Galla, Mark Girard, Clint Kirchhoff, Loi Le, Xin Ma, JJ Lubinski, Matt Maly, Mohsin Rahim, Dennis Ugolini, and Faizan Zubair.**