Problem of the Week

09/07/2009 to 09/18/2009

How many positive integers n are there such that n is an exact divisor of at least one of the following numbers:

$$10^{40},\ 20^{30}?$$

Solution: There are 2301 such divisors.

We first see that $10^{40} = 2^{40} \cdot 5^{40}$ and $20^{30} = 2^{60} \cdot 5^{30}$. This means that a positive integer divides 10^{40} if it is of the form $2^i 5^j$ where $0 \leq i, j \leq 40$, and so there are $41 \cdot 41 = 1681$ such divisors. Similarly, a positive integer divides 20^{30} if it is of the form $2^k 5^l$ where $0 \leq k \leq 60$ and $0 \leq l \leq 30$, so there will be $61 \cdot 31 = 1891$ such divisors. However, some of these divisors were counted twice, namely, any divisor of the form $2^s 5^t$ where $0 \leq s \leq 40$ and $0 \leq t \leq 30$, and there are $41 \cdot 31 = 1271$ such factors. Thus, there are

$$1681 + 1891 - 1271 = 2301$$

such integers n which divide at leat one of the given numbers.

Solutions for this problem were submitted by Quentin Funk, Matt Galla, Mark Girard, JJ Lubinski, Matt Maly, Jennifer Steele, David Stuck, Dennis Ugolini, Faizan Zubair.