Problem of the Week

10/19/2009 to 10/30/2009

How many terminal zeros are there at the end of the number 1000!? (Note: $x! = x \cdot (x-1) \cdots 2 \cdot 1$. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.)

Solution: The number of terminal zeros depends on the numbers of 10's which can be factored out of of 1000!. Since $10 = 2 \cdot 5$, we can see how many times 2 is a factor of 1000! as well as how many times 5, and then take the smaller of those two numbers to give the total number of factors of 10. Now, since $1000 = 200 \cdot 5$; there are 200 factors in 1000! which are divisible by 5, that is, 5 divides

$$5, 10, 15, 20, 25, \cdots, 995, 1000.$$

Now, there are another 40 factors of 5 in 1000! since $1000 = 40 \cdot 5^2$, namely, aside from the 200 5's already factored out, we can factor another 5 out of the terms

$$25, 50, 75, 100, \cdots, 975, 1000.$$

Similarly, since $1000 = 8 \cdot 5^3 = 1 \cdot 5^4 + 375$, there are 8 more factors of 5 corresponding to a 3rd power of 5 and 1 more for a 4th power of 5. Thus, the largest power of 5 which divides 1000! is

$$5^{200}5^{40}5^{8}5^{1} = 5^{249}.$$

Since 2 divides every other number in the product

$$1 \cdot 2 \cdot 3 \cdot 4 \cdots 998 \cdot 999 \cdot 1000 = 1000!,$$

there are at least 500 factors of 2 in 1000!. As $500 > 249$, there are 249 terminal zeros in 1000!.

Solutions for this problem were submitted by Matt Galla, Mark Girard, Santhana Krishnan (IIT, Bombay), Jeff Liese (Cal Poly, SLO), Matt Maly, Paul Nguyen, David Stuck, and Dennis Ugolini.