Problem of the Week #9
12/09/2019 to 1/12/2020

Each New Year’s Eve, Alice likes to say goodbye to the year by flipping a coin the number of times corresponding to the outgoing year. Her antagonistic brother, Bob, says he prefers to forget the past year by ringing in the new year, so he always flips a coin the number of times corresponding to the incoming year; moreover, Bob always makes a bet with Alice that he will flip more heads than she will. What is the probability that Bob wins the bet this year by flipping more heads in his 2020 tosses than Alice does in her 2019 tosses?

**Solution:** The answer is 1/2, and the key insight here is that because Bob has more tosses, he can flip more heads than Alice or more tails, but as he only has one more toss, he cannot do both.

So first fix Alice’s sequence of tosses. Since any sequence in which Bob flips more heads than Alice’s sequence is in one-to-one correspondence with a sequence in which he flips more tails—just switch in any sequence all the heads to tails and tails to heads—there must be the same number of each type of sequence. The probability of flipping a winning sequence must then be equal to that of flipping a losing sequence, thus probability of flipping either is 1/2.

**Solutions for this problem were submitted by** Otar Beridze (Georgia, the country), Phil Boyd (Manchester, England), Adam Cordeiro (Japan), T.J. Gaffney (Las Vegas, NV), Soham Ghosh (India), Rob Hill (Gambrills, Maryland), Lincoln James (Chicago, IL), Tengiz Kutchava (Georgia, the country), Manuel Guillermo Flota López (México), Yann Michel (Paris, France), Charlie Miller (London, UK), Benjamin Phillabaum (Bothell, WA), and Zurab Zakaradze (Georgia, the country).