Let $\mathbb{P}$ be the set of all points $(x, y)$ in the Cartesian plane where $x$ and $y$ are integers, and suppose we wish to color every point in $\mathbb{P}$ with one of the colors Red, White, and Blue. Is it possible to color these points such that both of the following conditions hold?

i. Each color occurs infinitely often in infinitely many lines which are parallel to the $x$-axis.

ii. No line in the plane—that is, not just those parallel to the $x$-axis—contains points of all three colors.

**Solution:** At first reading this may seem impossible, but indeed, such a coloring is possible. To see this, first color each point $(x, y)$ as follows:

1. $(x, y)$ is Red if $x$ and $y$ have the same parity (that is, they are both odd or both even);
2. $(x, y)$ is White if $x$ is odd and $y$ is even;
3. $(x, y)$ is Blue if $x$ is even and $y$ is odd.

Then, property i. is satisfied since infinitely many Red points occur in every line parallel to the $x$-axis, and infinitely many White and Blue points appear, respectively, in every other line parallel to the $x$-axis.

To show that property ii. is also satisfied, consider the Red point $(x_r, y_r)$, the White point $(x_w, y_w)$, and the Blue point $(x_b, y_b)$. Then by construction, $x_w - x_r$ and $y_w - y_r$ must have different parities, but $x_b - x_w$ and $y_b - y_w$ are both odd. This gives that

$$(x_w - x_r)(y_b - y_w) \neq (x_b - x_w)(y_w - y_r) \Rightarrow \frac{y_b - y_w}{x_b - x_w} \neq \frac{y_w - y_r}{x_w - x_r}.$$
Thus, these three points are not on the same line.

Solutions for this problem were submitted by Matthew A. Brom (Troy, NY), T.J. Gaffney (Las Vegas, NV), Ben Gustafson (TU), Rob Hill (Gambrills, Maryland), Lincoln James (Chicago, IL), Steve King (Pullman, WA), Yann Michel (Paris, France), and François Seguin (Amiens, France).