On a square board, divided into a $2021 \times 2021$ grid, we place 2021 rooks that cannot attack each other, i.e., no rook is in the same row or column as another. Then, each of the 2021 rooks makes exactly one knight’s move. Is it possible to do this in a way so that all of the rooks still cannot attack one another?

**Solution:** This is not possible.

Label the squares of the board in such a way that the cell in the $i$-th row and $j$-th column is labeled as $(i, j)$, and denote the 2021 cells that contain the initial placement of rooks by $(x_1, y_1), (x_2, y_2), \ldots, (x_{2021}, y_{2021})$. The condition that the 2021 rooks are non-attacking is equivalent to saying that $x_1, x_2, \ldots, x_{2021}$ and $y_1, y_2, \ldots, y_{2021}$ are each permutations of the numbers $1, 2, \ldots, 2021$. Defining $R$ to be the sum of the coordinates of any 2021 non-attacking rooks gives that

$$R = x_1 + x_2 + \cdots + x_{2021} + y_1 + y_2 + \cdots + y_{2021} = 2021 \cdot 2022 = 4082462,$$

and in particular we note that $R$ is even. Now see that if a rook placed at $(x_k, y_k)$ makes a knight’s move, then it’s new coordinate is one of the following:

$$(x_k \pm 1, y_k \pm 2) \text{ or } (x_k \pm 2, y_k \pm 1).$$

In any case, this move changes the total of $R$ by $\pm 1$ or $\pm 3$. Since there are an odd number of rooks, the 2021 knight’s moves from our initial placements would change $R$ by an odd amount, meaning that the sum of the coordinates of our new placement would also be odd, and therefore not 4082462. Thus, the rooks are no longer non-attacking.
(This will be the last problem of the school year, and we’ll start up again in August. Per usual, if you have submitted solutions this year, I will notify you via email when we begin anew. Until then, have a fun and healthy summer!)

Solutions for this problem were submitted by Phil Boyd (Manchester, England) M.V. Channakeshava (India) T.J. Gaffney (Las Vegas, NV) Rob Hill (Gambrills, MD) Hari Kishan (India) Tengiz Kutchava (Georgia, the country) Tin Lam (St. Louis, MO) Yann Michel (Paris, France) François Seguin (Amiens, France) Zurab Zakaradze (Georgia, the country).