## Problem of the Week #1

**8/29/2022 to 9/11/2022**

**The mythical *mathipede* grows at a rate of 1 meter per hour, and when they reach their maximum length of 1 meter, they stop growing. In addition, any full-grown mathipede may be cut into two parts totaling a length of 1 meter, and then those two mathipede parts immediately begin to grow at a rate of 1 meter per hour until fully grown. This process of dissection may be repeated on any full-grown mathipede. Is it possible, starting with one full-grown mathipede, to obtain ten full-grown mathipedes in less than an hour?**

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**Solution:** This is possible. First divide an hour into $2^{10} = 1024$ equal parts, which we'll call *mathiseconds*. We then have that a growing mathipede grows at $1/1024$ meters per mathisecond.

At time $t = 0$, make the first cut so that we cut off $1/1024$ of the mathipede. Then after one mathisecond, we have a full-grown mathipede, and the part that begins as $1/1024$-th of a meter will be full-grown with one mathisecond left in the hour. Indeed, if at $t_n = 2^n - 1$ mathiseconds ($n = 0, 1, 2 \ldots, 9$) we cut a full-grown mathipede into lengths $A_n = 2^n/1024$ meters and $B_n = 1 - 2^n/1024$ meters, then the one of length $A_n$ will always grow to full length with one mathisecond left in the hour, whilst the one of length $B_n$ will take always take $2^n/1024$ mathiseconds to grow back. Accordingly, making these cuts at the times $t_0, t_1, \ldots, t_9$ will yield, with 1 mathisecond left in the hour, 10 full grown mathipedes, namely, the ones that had starting lengths $A_0, A_1, \ldots, A_9, B_9$.

(*Note*: What we see from this argument is that there is nothing particularly special about wanting 10 full-grown mathipedes, and indeed, if we could cut fast enough, then in one hour we could obtain any number of mathipedes we desire.)

**Solutions for this problem were submitted by Matthew A. Brom (Albany, NY), Ritwik Chaudhuri (India), Rob Hill (Gambrills, MD), Hari Kishan (India), Lukas Klawuhn (Germany), Tengiz Kutchava (Georgia, the country), Jeff Liese (San Luis Obispo, CA), Yann Michel (Paris, France), Jorge Roberto Pech May (Campeche, México) Benjamin Phillabaum (Lafayette, IN), Luciano Santos (Lisboa, Portugal), François Seguin (Amiens, France), Dennis Ugolini (Trinity), and Steve Wiese (Austin, TX).**