



### Problem of the Week #3

9/26/2022 to 10/9/2022

Without the aid of a computational device, solve for  $x$  in the following equation:

$$4043 = x + \frac{x}{1+2} + \frac{x}{1+2+3} + \cdots + \frac{x}{1+2+3+\cdots+4043}.$$

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**Solution:** The solution is  $x = 2022$ . Begin by noting the denominators on the right can be rewritten as follows:

$$1+2+3+\cdots+k = \frac{(k+1)k}{2}.$$

Then by factoring  $x$  out of the righthand side, the equation we are asked to solve becomes

$$\begin{aligned} 4043 &= x \left( 1 + \frac{1}{\frac{3(2)}{2}} + \frac{1}{\frac{4(3)}{2}} + \cdots + \frac{1}{\frac{4044(4043)}{2}} \right) \\ &= 2x \left( \frac{1}{2(1)} + \frac{1}{3(2)} + \frac{1}{4(3)} + \cdots + \frac{1}{4044(4043)} \right) \\ &= 2x \left[ \left( 1 - \frac{1}{2} \right) + \left( \frac{1}{2} - \frac{1}{3} \right) + \left( \frac{1}{3} - \frac{1}{4} \right) + \cdots + \left( \frac{1}{4043} - \frac{1}{4044} \right) \right] \\ &= 2x \left( 1 - \frac{1}{4044} \right) = 2x \left( \frac{4043}{4044} \right). \end{aligned}$$

Solving this equation yields  $x = 2022$ .

**Solutions for this problem were submitted by Ziad Aramouni (Lebanon), Colin Bown (Austin, TX), M.V. Channakeshava (India), Ritwik Chaudhuri (India), Quentin Finn (alum), Evan Fu (Beaverton, OR), Amelia Gibbs (TU), Ong See Hai (Singapore), Rob Hill (Gambrills,**

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