



Problem of the Week #4

10/10/2022 to 10/23/2022

Consider a 2022×2022 grid in which each number from 1 to 2022^2 has been placed in a cell at random. Is it possible that no two cells which share a common side or a common vertex have entries with a sum that is divisible by 4?

Solution: This is not possible.

Begin by taking all of the entries modulo 4, leaving every cell with a 0, 1, 2, or 3. Since 2022^2 is divisible by 4, there are an equal number of cells containing each entry. Now divide the grid into 1011^2 disjoint 2×2 subgrids. If there were any subgrid with two 0's or two 2's, the sum of the corresponding entries in the original grid would be divisible by 4, and we'd be done, but as there are as many subgrids as 0's and 2's, it is possible that each subgrid contains exactly one 0 and exactly one 2. Similar logic tells us that if there were any subgrid with one 1 and one 3, the sum of the corresponding entries in the original grid would be divisible by 4. So, to avoid this each subgrid would have to contain two 1's or two 3's. But as there are an odd number of subgrids, this is impossible. Thus, some subgrid must contain two entries with a sum divisible by four.

Solutions for this problem were submitted by Ritwik Chaudhuri (India), Evan Fu (Beaverton, OR), Amelia Gibbs (TU), Ong See Hai (Singapore), Rob Hill (Gambrills, MD), Kipp Johnson (Beaverton, OR), Steve King (Pullman, WA), Hari Kishan (India), Tengiz Kutchava (Georgia, the country), Yann Michel (Paris, France), François Seguin (Amiens, France), Hicham Selmouni (Paris, France), and Zurab Zakaradze (Georgia, the country).