Chapter 0: 6, 7, 8, 14, 18, 19, 22, 28, 48-50

Extra Problems for HW #1

Problem 1: Suppose \( f : X \to Y \) and \( g : Y \to X \) such that \( f \circ g \) is one-to-one and \( g \circ f \) is onto. Show that \( f \circ g \) and \( g \circ f \) are bijections.  

Hint: You may first want to show that \( g \) is a bijection.

Problem 2: Prove that for any \( n \in \mathbb{N} \), \( n \) has a unique representation of the form \( n = c_11! + c_22! + \cdots + c_kk! \), for some \( k \geq 1 \), with \( 0 \leq c_j \leq j \) for each \( j \) and \( c_k \neq 0 \).  

Hint: You may use the following lemma: For all \( n \in \mathbb{N} \), \( n! - 1 = \sum_{m=1}^{n-1} m \cdot m! \).

Problem 3: Let \( \mathbb{R}^* = \mathbb{R} - \{0\} \). Show that \( \mathbb{R}^* \) is a group under multiplication.