HW #5, due February 20th

Chapter 5: 4, 8, 10, 13, 19, 21, 24

Extra Problems for HW #5

**Problem 1:** Prove the following lemma that was given in class: A cycle of length \( n \) has order \( n \).

**Problem 2:** Prove or disprove the following statement: \( S_6 \) contains a cyclic subgroup of order 8.

**Problem 3:** Find 8 subgroups of \( S_4 \) which are not \( e, S_4, D_4, \) or \( A_4 \).

**Problem 4:** Let \( X, Y \) be groups. We say that \( f : X \to Y \) is a group homomorphism if, given \( u, v \in X \), \( f(uv) = f(u)f(v) \). For any \( n \in \mathbb{N} \), define \( \text{sgn}: S_n \to \{\pm1\} \) by \( \text{sgn}(\sigma) = 1 \) if \( \sigma \) is even and \( \text{sgn}(\sigma) = -1 \) if \( \sigma \) is odd. Show that \( \text{sgn} \) is a homomorphism.