Problem 1: Let $G$ be an Abelian group and suppose that $G = H \times K$ for some $H, K \leq G$. Suppose $p$ is prime. Show that $G^p = H^p \times K^p$. Here, $G^n = \{x^n \mid x \in G\}$.

Problem 2: Find a ring, $R$, with unity and a subring $S$ of $R$ such that $S$ has a unity which differs from $R$.

Problem 3: Let $R$ be a ring and suppose $S$ and $T$ are subrings of $R$. Prove or disprove the following assertions.

(a) $S \cap T$ is a subring of $R$.

(b) $S \cup T$ is a subring of $R$.

Problem 4: Show that $\mathbb{Z}_n$ is an integral domain if and only if $n$ is prime.