Homework 3
Due Date: February 2

Read Chapter 4.
Chapter 2: 9, 29, 33, 38
Chapter 3: 4, 6, 8, 9, 14, 15, 26, 29, 34, 36

Extra Problems for HW #3

**Problem 1:** Prove or disprove that there is a group $G = \{e, a, b, c\}$ such that $|a| = |b| = 2$ and $|c| = 4$.

**Problem 2:** Let $H$ and $K$ be subgroups of a group $G$. Determine if the following are subgroups of $G$. If one is not a subgroup, then give a counterexample.
(i) $H \cap K$
(ii) $H \cup K$

**Problem 3:** Determine which of the following are subgroups of the given group $G$. If it is a subgroup, determine whether or not it is Abelian. For this problem, assume $p$ is a fixed prime, and if $S$ is any set, define $B(S)$ to be the set of all bijections from $S$ to itself.
(i) $\{ M \in GL(2, \mathbb{R}) \mid M^2 = I_2 \}; G = GL(2, \mathbb{R})$ ($I_2$ is the $2 \times 2$ identity matrix.)
(ii) $\{a/p^k \in \mathbb{Q} \mid a, k \in \mathbb{Z}, a \neq 0\}; G = (\mathbb{Q}^*, \cdot)$
(iii) $H_p = \{ a/b \in \mathbb{Q} \mid a, b \in \mathbb{Z} \text{ and } \gcd(b, p) = 1 \}; G = (\mathbb{Q}, +)$
(iv) $L = \{ g \in B(\mathbb{R}) \mid g(x) = ax + b, \ a \in \mathbb{Q}^*, \ b \in \mathbb{R} \}; G = (B(\mathbb{R}), \circ)$
(v) $F = \{ f \in B(\mathbb{N}) \mid f(n) = n \text{ for infinitely many } n \in \mathbb{N} \}; G = (B(\mathbb{N}), \circ)$

**Problem 4:** Let $G$ be a group and suppose that $H \leq G$. For $g \in G$, define $g^{-1}Hg = \{g^{-1}hg \mid h \in H\}$. Show that $g^{-1}Hg \leq G$.

**Bonus Problem #3:** Let $G$ be a group. Suppose that $a, b \in G$ such that $a$ and $b$ commute and their orders are relatively prime. Prove that $|ab| = |a||b|$. 