Read Chapter 8 and the proof of Theorem 7.3.

Chapter 5: 31

Chapter 7: 1, 2, 6-9, 12-16, 18-20, 22, 23, 25, 26, 28, 31, 33, 36, 40, 42, 49

Extra Problems for HW #8

**Problem 1:** We say that a subgroup $H$ of a group $G$ is a normal subgroup of $G$ if $aH = Ha$ for every $a \in G$, and we denote this by $H \triangleleft G$. Show that if $|G:H| = 2$, then $H \triangleleft G$.

**Problem 2:** For $n \in \mathbb{N}$, let $S_n$ denote the $n^{th}$ symmetric group and let $A_n$ denote the alternating group. Determine $|S_n : A_n|$.

**Problem 3:** Suppose $K = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $G = \{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \in GL(2, \mathbb{Z}) | ac = \pm 1 \}$, and $H = \{ M \in G | b = 0 \}$.

a.) Show that $H \leq G \leq GL(2, \mathbb{R})$.

b.) Show that $\langle K \rangle \triangleleft G$.

**Bonus Problem #5:** Suppose $G$ and $H$ are finite groups and let $G_k = \{ x \in G | |x| = k \}$ and $H_k = \{ x \in H | |x| = k \}$. If $|G_k| = |H_k|$ for every possible value of $k$, must it be the case that $G \approx H$?

**Bonus Problem #6:** Suppose $H \leq (\mathbb{Q}, +)$ under addition. Show that if $H \neq \mathbb{Q}$, then $|\mathbb{Q} : H| = \infty$. 