Homework 9  
Due Date: April 2

Book Problems for HW #9

Read Chapter 15.

Chapter 14: B2 (where $\mathcal{F}(\mathbb{R}) = \{ f : \mathbb{R} \to \mathbb{R} \}$), B3, B6, C1-4, D3, D6, E1, E5, F1, F5, G1, G2, H1-3

Supplementary Problems for HW #9

**Problem 1:** Let $f : \mathbb{Z}_{50} \to \mathbb{Z}_{15}$ be a homomorphism with $f(7) = 6$.

i. Determine a formula for $f(x)$.

ii. Determine $\text{Ker } f$.

iii. Determine the image of $f$.

iv. Determine $f^{-1}(3)$.

**Problem 2:** Let $G, G'$ be groups and suppose $f : G \to G'$ is a homomorphism onto $G'$. Prove that if $G'$ has an order of element $n$, then $G$ has an element of order $n$.

**Problem 3:** Let $G$ be a group with normal subgroups $H, K$ such that $K \leq H$. Show that $K \triangleleft H$.

**Problem 4:** Let $f : G \to G'$ be a group homomorphism with kernel $K$. If $H \leq G$, show that $f^{-1}(f(H)) = HK$. Use this to find an “if and only if” condition such that $f^{-1}(f(H)) = H$. 