\[ \text{HW \#8, due October 24}^{th} \]

Chapter 6: 40

Chapter 7: 7, 12, 16, 18-22, 24, 28, 31, 33, 36, 45

Extra Problems for HW \#8

\textbf{Problem 1:} Suppose \( G \) is a group and \( H \leq G \). Find an example where \( xH = yH \)
but \( Hx \neq Hy \).

\textbf{Problem 2:} Suppose \( K = \left( \begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right) \), \( G = \left\{ \left( \begin{array}{cc} a & b \\ 0 & c \end{array} \right) \in GL(2, \mathbb{Z}) \mid ac = \pm 1 \right\} \),
and \( H = \left\{ \left( \begin{array}{cc} a & 0 \\ 0 & c \end{array} \right) \in G \right\} \).

\begin{enumerate}
\item a.) Show that \( H \leq G \leq GL(2, \mathbb{Z}) \).
\item b.) Show that \( x(K) = \langle K \rangle x \) for every \( x \in G \).
\end{enumerate}

\textbf{Problem 3:} Let \( G \) be a group and suppose that \( H \leq K \leq G \). Show that \([G : H]\) is
finite if and only if both \([G : K]\) and \([K : H]\) are finite. (Hint: See Problem 27 in
Chapter 7.)

\textbf{Problem 4:} Let \( G \) be a group and suppose that \( H, K \leq G \). Show that if \([G : K]\) is
finite, then \([H : H \cap K]\) is finite.