Exam 1 Review Assignment, due Monday, September 24th (30 points)

1. (a) Evaluate \( \lim_{t \to 0} \left( \frac{1}{t^{1/4} + 1} - \frac{1}{t} \right) \).
(b) Evaluate \( \lim_{x \to 2} \frac{x^4 - 2x^3 + 3x^2 - 15x + 18}{x^3 - 8} \).
(c) Evaluate \( \lim_{x \to 3} \frac{\sqrt[3]{7} - x}{\sqrt[3]{4} - x - 1} \).
(d) Evaluate \( \lim_{\theta \to 0} \theta^4 \cot^3(2\theta) \csc(3\theta) \).

2. Suppose that \( f(x) = x^3 \sin(1/x) \) for \( x \neq 0 \) and \( f(0) = 0 \). Calculate \( f'(0) \) if it exists.

3. Use the definition of the derivative to find \( g'(x) \) if \( g(x) = \frac{x}{\sqrt{x} + 1} \).

4. Let \( p(t) = t^2 - c^2 \) for \( t < 3 \) and \( p(t) = 2ct + 2 \) for \( t \geq 3 \). Find all values of \( c \) that make \( p(t) \) continuous on \( (-\infty, \infty) \).

5. Find all points on the graph of \( f(x) = 2x^3 + 3x^2 - 12x + 1 \) where the tangent line is horizontal.

6. Compute the following derivatives.
   (a) Find \( f'(-1) \) if \( f(x) = (x^2 + 4x)^2 (x^3 - 1)^3 (x^4 + x) \).
   (b) Find \( h'(0) \) if \( h(q) = \frac{(q^3 + 3q + 1)^{\frac{3}{2}}}{q^{1/2}} \).

7. Suppose that \( g(2) = 4 \) and \( g'(2) = -3 \).
   (a) Find an equation of the line tangent to the graph of \( g(x) \) at the point \( (2, 4) \).
   (b) Compute \( \lim_{x \to 2} \frac{g(x) - g(2)}{x - 2} \).

8. Find an equation of the line tangent to the curve \( y = (\frac{x^2 + 1}{x^2 - 1})^2 \) at the point \( (2, \frac{25}{9}) \).

9. Suppose \( d(x) = x^4 - 4x^2 + 2 \). Find the absolute maximum and minimum values of \( d(x) \) on \([-3, 1]\).

10. Find the area of the largest rectangle which can be inscribed in the ellipse \( \frac{x^2}{9} + \frac{y^2}{36} = 1 \).

11. Problem number 86 on page 214

12. A ball is thrown vertically upward at time \( t = 0 \) with an initial velocity of 100 ft/sec at an initial height of 84 feet. This gives the ball a height function of \( h(t) = -16t^2 + 100t + 84 \).
   (a) Find the maximum height of the ball.
   (b) Find the velocity of the ball when it hits the ground.