Exam 1 Review Assignment, due Wednesday, October 1^{st} (30 points)

1. Find the domain of $f(x) = \sqrt{\dfrac{x^2 - 10x - 24}{x^2 - 1}}$.

2. (a) Evaluate $\lim\limits_{t \to 0} \left(\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t} \right)$.

(b) Evaluate $\lim\limits_{x \to 2} \dfrac{x^4 - 2x^3 + 3x^2 - 15x + 18}{x^3 - 8}$.

(c) Evaluate $\lim\limits_{x \to 3} \dfrac{\sqrt{7-x} - 2}{\sqrt{4-x} - 1}$.

(d) Evaluate $\lim\limits_{t \to \infty} \dfrac{\sin(t^2+1)}{1-t}$.

(e) Evaluate $\lim\limits_{x \to \infty} \left(\dfrac{x^2}{x+2} - \dfrac{x^3}{x^2+3} \right)$.

3. Let $p(t) = t^2 - c^2$ for $t < 3$ and $p(t) = 2ct + 2$ for $t \geq 3$. Find all values of c that make $p(t)$ continuous on $(-\infty, \infty)$.

4. Find all horizontal asymptotes of the graph of $r(t) = \dfrac{4t^2 - 6t + 7}{\sqrt[5]{32t^{10} + t^5 - 7}}$.

5. Use the definition of the derivative to find $g'(3)$ if $g(x) = \dfrac{x}{\sqrt{x+1}}$.

6. Find all points on the graph of $f(x) = 2x^3 + 3x^2 - 12x + 1$ where the tangent line is horizontal.

7. Compute the following derivatives.

(a) Find $f'(0)$ if $f(x) = (x^2 + 4x + 1)^2(x^3 - 1)^{10}e^{2x}$.

(b) Find $h'(0)$ if $h(q) = \dfrac{(e^{q^2+q} - 2)^8}{\sin^2(q) + 3}$.

(c) Find $x'(2)$ is $x(t) = \sqrt{\sqrt{\sqrt{t^2 + 3t + 6}}}$.

(c) Find y' if $y(t) = \cos^3(\sec(t - 1/t))$.

8. Suppose that $g(2) = 4$ and $g'(2) = -3$.

(a) Find an equation of the line tangent to the graph of $g(x)$ at the point $(2, 4)$.

(b) Compute $\lim_{x \to 2} \dfrac{g(x) - g(2)}{x - 2}$.

9. Find an equation of the line tangent to the curve $y = \left(\dfrac{x^2 + 1}{x^2 - 1}\right)^2$ at the point $\left(2, \frac{25}{9}\right)$.

10. Suppose that $f(x) = x^2 sin(1/x)$ for $x \neq 0$ and $f(0) = 0$. Calculate $f'(0)$ if it exists.

11. Find an equation of the line tangent to the graph of the curve $\sin(xy) = \cos(x + y)$ at the point $(\frac{\pi}{2}, 0)$.

12. Section 3.5, problem #66.

13. A ball is thrown vertically upward at time $t = 0$ with an initial velocity of 100 ft/sec at an initial height of 84 feet. This gives the ball a height function of

$$h(t) = -16t^2 + 100t + 84.$$

(a) Find the maximum height of the ball.

(b) Find the velocity of the ball when it hits the ground.