Exam 3 Review Assignment, due Wednesday, December 3rd (30 points)

1. A particle is released from the point \((-1, 0)\) at time \(t = 0\) with an acceleration of \(\sin(t) + \cos(t)\) units/s\(^2\). Suppose the particle is initially moving to the left at 1 units/s.
   (a) Find the position of the particle after \(\pi\) seconds.
   (b) Find the total distance traveled by the particle in \(0 \leq t \leq \pi\).

2. Calculate the following limits.
   (a) \(\lim_{x \to 1} \frac{\sqrt{2x} - x^4 - \sqrt{x}}{1 - \sqrt{x}}\)
   (b) \(\lim_{y \to \infty} \frac{\ln(y)}{\sqrt{y} + \sqrt[3]{y}}\)
   (c) \(\lim_{x \to 0^+} (e^{1/x} - 1) \tan(x)\)
   (d) \(\lim_{x \to \infty} \left( \frac{x^2}{x + 2} - \frac{x^3}{x^2 + 3} \right)\)

3. For nonzero constants \(a\) and \(b\), calculate \(\lim_{x \to \infty} \left( 1 + \frac{a}{x} \right)^{bx}\).

4. Calculate the \(L_4\), \(R_4\), and \(M_4\) for \(h(x) = x^3 + x + 1\) on \([-2, 6]\).

5. Express \(\lim_{n \to \infty} \sum_{i=1}^{n} \frac{\pi}{3n} \ln \left( \cos \left( 2\pi + \frac{\pi i}{3n} + e^{\pi i} + \frac{3n}{\pi i} \right) \right)\) as a definite integral.

6. (a) Find \(g(x)\) if \(g'(x) = x\sqrt{x} + 1\) and \(g(4) = 7\).
   (b) Find \(p(t)\) if \(p''(t) = e^{3t} - \sin(2t), p'(0) = 1,\) and \(p(0) = -2\).

7. Evaluate \(\lim_{x \to 0^+} \int_{0}^{x} \frac{t^2 \csc^2(t) dt}{5x}\).

8. Find \(g'(x)\) if \(g(x) = \int_{\sin(x)}^{3x + 1} \frac{\tan^2(2t)}{1 + t \cos(t^2)} dt\).

9. A farmer has 400 feet of fencing to construct a rectangular corral. One side of the fence will be built along a wall which is 100 feet long, and the farmer can use some or even all of this wall as one side of his corral. What is the maximum area the farmer can enclose in the corral.

10. Find an equation of the line through the point \((3, 5)\) that cuts off the least area from the first quadrant.

11. Use Newton’s method to find the absolute maximum value of \(h(t) = t - t^2 + \cos(t)\) correct to eight decimal places.
12. Compute the following integrals.

(a) \( \int \sin^2(u) \cos^2(u) \, du \)

(b) \( \int_{0}^{\pi/4} \sin^3(t) \cos^3(t) \, dt \)

(c) \( \int_{0}^{1} x^3 \sqrt{x^2 + 3} \, dx \)

(d) \( \int_{-2}^{2} (3 + \sqrt{4 - t^2}) \, dt \)

(e) \( \int_{-3}^{2} |x^3 + x^2 - 2x| \, dx \)

(f) \( \int \frac{\ln(\ln(x))}{x \ln(x)} \, dx \)

(g) \( \int_{1}^{2} \frac{\sqrt{t^2} - \sqrt{t} + t^3}{2t^2} \, dt \)

13. A large rock is dropped from the top of a 128 foot tall building. At the same time, a ball is thrown straight upwards from ground level from a point directly below the rock. With what initial velocity should the ball be thrown so that the ball meets the rock when the rock is halfway, in terms of height, to the ground?

14. When a mass is hung from a spring and then displaced from its natural hanging position, the spring, and consequently the mass, will oscillate (up and down). In this case, the acceleration of the mass at time \( t \), in seconds, is given by

\[
a(t) = -\frac{4\pi^2 x_m \cos(2\pi t/T)}{T^2},
\]

where \( T \) is the period of motion and \( x_m \) is the maximum displacement of the mass from its natural hanging position. If the initial velocity of the mass is \( v_0 \) and initial position is \( y_0 \), find the velocity and position of the mass at any time \( t \).

15. For the following problems, sketch the area bounded by the given curves, and then compute this area.

(a) \( x = y^3 - y, \ x = 1 - y^4 \)

(b) \( y = x^3, \ y = 2x^3 + x^2 - 2x \)