Section 3.9: Related Rates

How do we solve a related rates problem?

1. Draw a good picture.
   (a) Label anything which changes with a variable.
   (b) Label the rates of changes of those variables.
2. Define all of the variables from the picture.
3. Find an equation which relates all of the variables to one another.
4. Differentiate both sides of the equation and solve.
5. Plug in the values at the specific time we are interested in.
6. Answer the question.

Example 1: A lighthouse is 6 km offshore from the nearest point P on a vertical shoreline. If the lighthouse’s light makes 4 revolutions per minute, how fast is the beam of light moving along the shoreline when it is 2 km from P?

Example 2: A 12 foot long ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 3 ft/sec., how fast is the angle between the wall and the ladder changing when the angle is 30°?

Example 3: Two sides of a triangle, x and y, are separated by an angle of 30°. If the length of x increases at a rate of 2 cm/sec and the length of y decreases at a rate of 3 cm/sec, find the rate at which the area of the triangle decreases when x = 10 cm and y = 7 cm.

Example 4: At noon, ship A is 100 kilometers west of ship B. If ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h, how fast is the distance between them changing at 4:00 p.m. later that day?

Example 5: Water leaks out of an inverted conical tank at a rate of 10000 cm³ per minute at the same time that water is being pumped into the tank at some constant rate. If the tank has a height of 6 m and a diameter of 4 m at the top and if the water level is rising at a rate of 20 cm per minute when the height of the water is 2 m, find the rate at which water is being pumped into the tank.