Exam 2 Review Assignment
Due Date: Thursday, November 19
30 points

Problem 1. Let \( f(x) = \sqrt{2 + x} \).

i. Find the linearization of \( f(x) \) near \( x = 30 \).

ii. Use (i) to estimate \( \sqrt{31} \).

Problem 2. Let \( y = e^{x^2 - 4\cos(\pi x)} \).

i. Compute the differential, \( dy \).

ii. Evaluate the differential found in (i) if \( x = 2 \) and \( dx = .2 \).

Problem 3. Calculate the following limits.

i. \( \lim_{x \to 1} \frac{x^6 - 4x^5 - 10x^4 + 24x^3 + 13x^2 - 44x + 20}{x^7 + 4x^6 - 9x^5 - 34x^4 + 47x^3 + 72x^2 - 135x + 54} \)

ii. \( \lim_{x \to 0^\ast} (e^{1/x} - 1) \tan(x) \).

Problem 4. A person 6 feet tall walks at 5 feet per second along one edge of a road 30 feet wide. Across the street, on the other edge of the road, is a light pole that is 18 feet tall. How fast is the length of the person’s shadow (on the ground) increasing when the person is 40 feet from the point directly across the street from the pole, walking with the pole at their back?

Problem 5. Suppose \( d(x) = x^4 - 4x^2 + 2 \). Find the absolute maximum and minimum values of \( d(x) \) on \([-3, 1]\).
Problem 6. Let \( f(x) = 5x^{2/3} - x^{5/3} \). Show that \( f \) satisfies the hypotheses of the Mean Value Theorem on \([0, 5]\), and find all numbers \( c \) which satisfy the conclusion of the Mean Value Theorem for \( f \) for this same interval.

Problem 7. Let \( p(x) = 5x^3 - 3x^5 \).

(a) On what interval(s) is \( p(x) \) increasing? decreasing?

(b) Classify all local extrema of \( p(x) \).

(c) On what interval(s) is the graph of \( p(x) \) concave up? concave down?

(d) Find all inflection points of the graph of \( p(x) \).

Problem 8. Find and classify all local extrema of \( r(t) = \frac{3}{5}t^5 - \frac{1}{2}t^4 - 2t^3 \).

Problem 9. A power line runs in an east-west direction through the countryside. Two new buildings will be built, and they will draw power from this power line by running lines from a single point on the existing line to each of the new buildings. If one building lies 6 miles south of the existing line and the other building lies 10 miles to the east and 2 miles south of the other other building, where on the existing line should the connections to these buildings be made in order to minimize the amount of cable used?

Problem 10. Find an equation of the line through the point \((3, 5)\) that cuts off the least area from the first quadrant.

Problem 11. Find the general antiderivative of \( g(t) = \frac{\sqrt[5]{t^3} - \sqrt{t} + t^3}{2t^2} \).

Problem 12. A large rock is dropped from the top of a 128 foot tall building. At the same time, a ball is thrown straight upwards from ground level from a point directly below the rock. With what initial velocity should the ball be thrown so that the ball meets the rock when the rock is halfway, in terms of height, to the ground?