Section 4.2: The Mean Value Theorem
Discussion Date: November 3

**Rolle’s Theorem:** Suppose that $f(x)$ is continuous on $[a, b]$ and differentiable on $(a, b)$. If $f(a) = f(b)$, then $f'(c) = 0$ for some number $c$ in $(a, b)$.

“Proof:”

**Mean Value Theorem:** Suppose that $f(x)$ is continuous on $[a, b]$ and differentiable on $(a, b)$. Then there exists some number $c$ in $(a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

“Proof:”
Four Consequences of the Mean Value Theorem:

To begin, suppose that $f(x)$ is continuous on $[a, b]$ and differentiable on $(a, b)$.

1. If $f'(x) = 0$ for every $x$ in $(a, b)$, then $f(x)$ is a constant for every $x$ in $[a, b]$.

2. If $f'(x) = g'(x)$ for every $x$ in $(a, b)$, then $f(x) = g(x) + C$ (with $C$ constant) for every $x$ in $[a, b]$.

3. If $f(x) = 0$ has distinct solutions, say $a$ and $b$, then $f'(c) = 0$ for some $c$ in $(a, b)$.

4. If $f'(x) > 0$ for every $x$ in $(a, b)$ then $f$ is an increasing function on $(a, b)$. If $f'(x) < 0$ for every $x$ in $(a, b)$ then $f$ is an decreasing function on $(a, b)$.

**Definition:** The function $f(x)$ is increasing on $(a, b)$ if $f(x_1) < f(x_2)$ for every $x_1, x_2$ in $(a, b)$ whenever $x_1 < x_2$. The function $f(x)$ is decreasing on $(a, b)$ if $f(x_1) > f(x_2)$ for every $x_1, x_2$ in $(a, b)$ whenever $x_1 < x_2$. 