Exam 1 Review Assignment, due Monday, September 28th (30 points)

1. Consider the inequalities \( x - 2y^2 \geq 0 \) and \( 1 - x - |y| \geq 0 \). Sketch the region in the \( xy \)-plane defined by these inequalities and find the area of this region.

2. The integral \( \int_0^\pi 2\pi (4 - x) \sin^4(x) \, dx \) represents the volume of a solid. Describe the solid. (Do not compute this integral.)

3. Find the volume of the solid generated by rotating the region bounded by the curves \( x = y^2 + 1 \) and \( x = 9 - y^2 \) about
   
   a.) the line \( x = 10 \).  
   b.) the line \( y = 3 \).

4. Find the volume of the solid generated by rotating the loop of the curve \( y^2 = x^2(x + 3) \) about
   
   a.) the \( y \)-axis.  
   b.) the line \( y = 3 \).

5. Consider the region \( \mathcal{R} = \{(x, y) \mid x \geq 1, 0 \leq y \leq 1/x\} \). Find the volume of the solid generated by rotating this region about the \( x \)-axis.

6. Evaluate \( \int \sin^3(2x) \cos^2(x) \, dx \).

7. Evaluate \( \int_0^{1/2} \sqrt{1 - 4y^2} \, dy \).

8. Evaluate \( \int_0^1 x^2 \sin(x^3) \, dx \).

9. Evaluate \( \int_1^4 e^{x^2} \, dx \).

10. Evaluate \( \int \frac{8x^4 - 12x^3 - 4x^2 + 2x - 6}{x^5 - x^4 - x + 1} \, dx \).

11. Evaluate \( \int te^t \cos(t) \, dt \).

12. Evaluate \( \int_0^{\pi/4} \tan^{5/2}(x) \sec^4(x) \, dx \).

13. Evaluate \( \int \frac{dw}{w^4\sqrt{w^2 - 4}} \).

14. Evaluate \( \int_{-2}^0 \frac{dx}{\sqrt{x^2 + 4x + 8}} \).

15. Evaluate \( \int \left( \frac{1}{5}x^5 + x^4 - 3x^2 \right) \sin(-2x) \, dx \).

16. Evaluate \( \int_1^\infty \frac{\ln(x)}{x^{3/2}} \, dx \).

17. Evaluate \( \int_{-\infty}^\infty \frac{x^2}{x^6 + 6x^3 + 10} \, dx \).