Exam 2 Review Assignment, due Wednesday, April 2nd (30 points)

1. Let \( \mathbf{u} = \langle 6, 2, 3 \rangle \) and \( \mathbf{v} = \langle 2, -3, 1 \rangle \).
   (a) Compute \( 4\mathbf{u} - 3\mathbf{v} \).
   (b) Find the length of \( \mathbf{u} \).
   (c) Find a unit vector which points in a direction opposite \( \mathbf{u} \).
   (d) Are \( \mathbf{u} \) and \( \mathbf{v} \) parallel, perpendicular, or neither? If your answer is
      neither, then find the angle between these vectors.
   (e) Find a vector, \( \mathbf{w} \), which is orthogonal to both \( \mathbf{u} \) and \( \mathbf{v} \).

2. Consider the points \( P(2, 5, 5) \) and \( Q(-6, 3, 1) \).
   (a) Find an equation for the line between \( P \) and \( Q \).
   (b) Find an equation of the plane consisting of all points which are
      equidistant from \( P \) and \( Q \).

3. Do Problem 54 in Section 12.3.

4. Consider the sequence \( \{a_n\}_{n \geq 1} \), where \( a_n = n \sin \left( \frac{\pi}{n} \right) \). Does this
   sequence converge to a real number \( L \)? If so, find \( L \).

5. Consider the sequence \( \{b_n\}_{n \geq 1} \), where \( b_n = \left( \frac{n^2 + 3}{n^2} \right)^{-2n^2+7} \). Does
   this sequence converge to a real number \( L \)? If so, find \( L \).

6. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. Make sure to clearly state
   which test you use.
   (a) \( \sum_{n=1}^{\infty} \frac{(n^2 + n + 6)^{7/3}}{(n^3 + 3n^2 + 3n + 1)^{7/4}} \)
   (b) \( \sum_{n=3}^{\infty} \frac{(-1)^n(n - 2) \ln(n)}{n^{3/2}} \)
   (c) \( \sum_{n=1}^{\infty} (n - 1)^2e^{-n} \)
   (d) \( \sum_{n=1}^{\infty} \frac{(-2)^{3n}}{n^n} \)
   (e) \( \sum_{n=1}^{\infty} (-1)^{n+3}2^{1/n^2} \)
   (f) \( \sum_{n=1}^{\infty} \frac{(-1)^{2n+1}n!}{(1)(5)(9)(13) \cdots (4n - 3)} \)
7. Compute the value of the series \( \sum_{n=1}^{\infty} \frac{2^{3n}}{(-3)^{2n+1}} \).

8. With proof, determine whether or not the following series is convergent or divergent.

\[
1 + \frac{1}{2} \left( \frac{19}{7} \right) + \frac{2!}{3^2} \left( \frac{19}{7} \right)^2 + \frac{3!}{4^3} \left( \frac{19}{7} \right)^3 + \frac{4!}{5^4} \left( \frac{19}{7} \right)^4 + \cdots
\]

9. Find the interval of convergence of each of the following series.

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^n n! x^{3n}}{20^n} \)

(b) \( \sum_{n=10}^{\infty} \frac{(-1)^n (2x - 1)^{2n+1}}{n \ln(n) [\ln(\ln(n))] } \)

(c) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)! 2^{2n+1}} \)

10. Use a power series to compute the following integral:

\[
\int \frac{e^x - 1}{x} \, dx.
\]

(Note: Your answer will be an infinite sum.)

11. Let \( f(x) = x^{50} - 3x^{30} + x^{10} - 2x^3 + 17 \).

(a) Find the 3\textsuperscript{rd}-degree Taylor polynomial, \( T_3(x) \), for \( f(x) \) at \( a = 1 \).

(b) Find the Maclaurin series of \( f(x) \).

(Hint: Think about this before you do it, although I guess you should do that on every problem.)

12. Evaluate \( \int_{1}^{\infty} \frac{\ln(x)}{x^{5/2}} \, dx \).