HW #3, due September 17th

Chapter 3: 1, 2, 4, 6, 8-10, 13-15, 19, 28

Extra Problems for HW #3

**Problem 1:** A cafeteria cook needs to stock 20 cans of generic sloppy joe mix on 3 different shelves.

(i): How many ways can this be done?

(ii): What if each shelf must contain at least one can of mix?

**Problem 2:** Give combinatorial proofs of the following facts:

(i): \( \sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0 \). (Hint: Use a problem from class.)

(ii): \( \sum_{k=0}^{n} k \binom{n}{k} = n2^{n-1} \).

**Problem 3:** Let \( X = \{x_1, x_2, \ldots, x_{20}\} \). Determine the number of ordered triples \( (S_1, S_2, S_3) \) such that \( S_1, S_2, S_3 \) partition \( X \). (Note: This allows for the possibility that \( S_i \) could be empty.)

*Bonus Problem:* Let \( X = \{x_1, x_2, \ldots, x_{20}\} \). Determine the number of ordered triples \( (S_1, S_2, S_3) \) such that

\[
S_1 \cup S_2 \cup S_3 = X,
\]

and

\[
S_1 \cap S_2 \cap S_3 = \emptyset.
\]