Extra Problems for HW #6

Problem 1: A six-sided die in which the faces are numbered 1, 2, 3, 4, 5, 6 is rolled ten times and the numbers which come up are recorded in sequence. How many ways are there to roll the die so that all six sides appear at least once? Notice here that the sequence of rolls corresponding to 1234561234 is different than that which corresponds to 1122334456.

Problem 2: Suppose we roll ten six-sided die simultaneously and record all the numbers which appear in no particular order. How many ways are there to roll the ten die so that all six numbers appear at least once?

Problem 3: Show that \( D_n = |B| \), where
\[
B = \{ X \in S_n \mid \text{for each } i \in [n] \exists \text{ a unique } j_i \in [n] \text{ such that } X(i) \neq j_i \}.
\]

Problem 4: Let \( S = \{2, 3, 5, 7\} \). How many of the integers from 1 to 10000 inclusive are divisible by exactly two of the elements of \( S \).

Problem 5: The eight cells of the brick below are to be colored with 2 red cells, 2 blue cells, 2 yellow cells, and 2 green cells.

(a) How many colorings are possible if no two consecutive cells have the same color?

(b) How many colorings contain 2 consecutive red cells and 2 consecutive green cells, but not consecutive blue or yellow cells?

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