Problem 1: Let $X = \{x_1, x_2, \ldots, x_{20}\}$. Determine the number of ordered triples $(S_1, S_2, S_3)$ such that $S_1, S_2, S_3$ form a set partition of $X$. (Note: This allows for the possibility that $S_i$ could be empty.)

Problem 2: Suppose we want to choose a committee of $k$ people from a group of $n$ people, where one of the $k$ members of the committee will be elected president. Find two different ways to compute the number of such $k$-person committees.

Problem 3: A college needs to place 200 incoming students - 110 female and 90 male - into 200 rooms of 3 different dorms. Supposing that Dorm A holds 80 students, Dorm B holds 70 students, and Dorm C holds 50 students, answer the following questions.

i. How many ways can these 200 students be placed into Dorms A, B, and C?

ii. If Dorm A is all-female, Dorm B is all-male, and Dorm C is co-ed, how many ways can these 200 students be placed into these dorms?

Problem 4: Suppose 10 indistinguishable seats are evenly spaced around a circular table. If 10 people wish to sit around the table, how many possible seating arrangements are there if two of the people do not wish to sit directly across from one another?

Problem 5: Suppose we wish to place rooks on an $8 \times 8$ chessboard such that no two rooks lie in the same row or column.

i. In how many ways can we place 8 indistinguishable rooks on the chessboard?

ii. What if the rooks are all distinct?

iii. What if there are 3 blue rooks, 4 red rooks, and 1 green rook?

iv. What if we are only placing 6 rooks and they are all distinct?

v. What if we want to place 5 indistinguishable rooks, and neither the top row nor the leftmost column is empty?

Problem 6: An amusement park ride has 6 different cars each of which holds 4 people and 24 people are ready to ride. There are no seats in each car, so the order in which people are placed in the individual cars is irrelevant, but the cars are ordered from front to back.

i. How many ways can the 24 riders be placed?

ii. How many ways can the 24 riders be placed if there are two people who do not wish to be in the same car as one another?

Problem 7: Twenty identical matchsticks are lined up in a row as follows,

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i. How many ways are there to choose 6 matchsticks?

ii. How many ways are there to choose 6 matchsticks if no 2 of the chosen sticks can be consecutive?

iii. How many ways are there to choose 6 matchsticks if there must be at least 2 sticks between each pair of chosen sticks?