Problem 1: Suppose that for some $k \geq 0$ you start on Pascal’s Triangle at the point $P_{k,k}$ and begin summing consecutive values down that column, stopping after some finite number of additions. For example, you could add $P_{1,1}$, $P_{2,1}$, and $P_{3,1}$ to get 6. Conjecture a formula for this sum and find a combinatorial proof of this formula.

Problem 2: Let $n \in \mathbb{N}$, and define an even (odd) set to be a set with an even (odd) number of elements. Give a combinatorial proof to show that exactly half of the subsets of an $n$-element set are even. Use this fact to give a combinatorial proof of the following identity:

$$\sum_{i=0}^{n} (-1)^n \binom{n}{i} = 0.$$

Problem 3: Suppose that for some $n \geq 0$ you start on Pascal’s Triangle at the point $P_{n,0}$ and begin summing consecutive values which are one cell to the right and one cell down, stopping after some finite number of additions. For example, you could add $P_{1,0}$, $P_{2,1}$, and $P_{3,2}$ to get 6. Conjecture a formula for this sum and find a combinatorial proof of this formula.