In-Class Problems
Date: September 8 and 10

Problem 1. A college needs to place 200 incoming students - 110 female and 90 male - into 200 rooms of 3 different dorms, A, B, and C. Suppose that Dorm A holds 80 students, Dorm B holds 70 students, and Dorm C holds 50 students.

i. How many ways can these 200 students be placed into Dorms A, B, and C?

ii. If Dorm A is all-female, Dorm B is all-male, and Dorm C is co-ed, how many ways can these 200 students be placed into these dorms?

Problem 2. Suppose a round table has 24 chairs which are indistinguishable from each other. How many ways could 24 people sit at the table? What if \( n \) people were seated at a round table with \( n \) indistinguishable seats? (For example, there are two ways for three people to sit at such a table.)

Problem 3. Suppose 10 indistinguishable seats are evenly spaced around a circular table. If 10 people wish to sit around the table, how many possible seating arrangements are there if two of the people do not wish to sit directly across from one another?

Problem 4. A man lives in a town in which all streets form a grid of city blocks of equal size. Each day the man walks to work, which lies seven blocks north and six blocks east of his home.

i. How many ways can the man walk to work in the morning if he walks exactly 13 blocks?

ii. How many ways can the man walk to and from work if he walks exactly 26 blocks and wants to always take a different path on his trip back home than the one he took to work in the morning?

iii. Suppose that there is construction on the block whose southwest corner lies three blocks east and two blocks north of the man's home. If that construction closes off the south side of that block to pedestrian traffic, how many ways can the man walk to work if he walks exactly 13 blocks?
Problem 5. Company XYZ has 2200 male and 1800 female employees. This company is going to select 100 of these employees to send to a week-long charity function in the Bahamas. If this group of 100 employees must contain at least 2 men, how many possible ways can Company XYZ choose these 100 employees?

Problem 6. Twenty identical matchsticks are lined up in a row as follows,

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i. How many ways are there to choose 6 matchsticks?

ii. How many ways are there to choose 6 matchsticks if no 2 of the chosen sticks can be consecutive?

iii. How many ways are there to choose 6 matchsticks if there must be at least 2 sticks between each pair of chosen sticks?

Problem 7. Suppose we wish to place rooks on an 8 \times 8 chessboard such that no two rooks lie the same row or column.

i. In how many ways can we place 8 indistinguishable rooks on the chessboard?

ii. What if the rooks are all distinct?

iii. What if there are 3 blue rooks, 4 red rooks, and 1 green rook?

iv. What if we are only placing 6 rooks and they are all distinct?

Problem 8. Let \( X = \{x_1, x_2, \ldots, x_{20}\} \). Determine the number of ordered triples \( (S_1, S_2, S_3) \) such that

\[
S_1 \cup S_2 \cup S_3 = X,
\]

and

\[
S_1 \cap S_2 \cap S_3 = \emptyset.
\]
**Problem 9.** Suppose we have 10 identical widgets, 1 whatsit, and 1 woodad. In how many ways can we distribute these 12 toys to 3 children if the whatsit and the woodad must go to different children and every child must get at least one toy?

**Problem 10.** How many permutations are there for letters of the word MISSISSIPPI?

**Problem 11.** An amusement park ride has 6 different cars each of which holds 4 people and 24 people are ready to ride. There are no seats in each car, so the order in which people are placed in the individual cars is irrelevant, but the cars are ordered from front to back.

i. How many ways can the 24 riders be placed?

ii. How many ways can the 24 riders be placed if there are two people who do not wish to be in the same car as one another?

**Problem 12.** Consider 5-card poker hands dealt from a standard 52-card deck. How many hands are there of each of the following rank:

i. Full house?

ii. Flush?

iii. Straight?

iv. Straight flush?

v. Two pair?