Problem 1. Suppose a group of students takes a survey about sports they like to play. Fifteen students responded that they like to play beach volleyball, 13 students responded that they like to play tennis, and 19 students said they like to play frisbee. Additionally, we find out that 6 students like to play both beach volleyball and tennis and 4 students like to play both beach volleyball and frisbee, while 8 students like to play both tennis and frisbee. Finally, 3 students like to play all three of the sports. Assuming that everyone in this group likes to play at least one of these three sports, how many students took the survey?

Problem 2. Recall from class the definition of the $n$-th derangement number, $D_n$.

i. For $n \in \mathbb{N}$, show that $D_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$.

ii. Use part i. to prove that $\lim_{n \to \infty} \frac{D_n}{n!} = \frac{1}{e}$.

iii. Use part i. to prove that for $n \geq 2$, $D_n = nD_{n-1} + (-1)^n$. 
**Problem 3.** Consider a $6 \times 1$ board colored with exactly 3 red cells and 3 blue cells.

i. Find the number of colorings of this board and list them all out. How many colorings exist such that no consecutive cells are colored identically? How many colorings exist such that exactly one color appears consecutively in the coloring? How many colorings exist such that both colors appear consecutively?

ii. Use the Principle of Inclusion/Exclusion to verify that there are exactly two colorings such that no consecutive cells are colored identically.

iii. From the calculations you did in part ii., define $|S| = \alpha_0$ and then define

\[
\alpha_k := \sum_{1 \leq a_1 < a_2 < \cdots < a_k \leq m} |A_{a_1} A_{a_2} \cdots A_{a_k}|,
\]

that is, each $\alpha_j$ is a term in the I/E sum without the +/- signs. Now define $\beta_k$ by

\[
\sum_{i=0}^{m} \alpha_i (t - 1)^i = \sum_{i=0}^{m} \beta_i t^i.
\]

Make a conjecture about what $\beta_k$ counts.

**Problem 4.** An $8 \times 1$ board is to be colored with 2 red cells, 2 blue cells, 2 yellow cells, and 2 green cells. How many colorings contain 2 consecutive red cells and 2 consecutive green cells, but not consecutive blue or yellow cells?