**HW #2, due March 28th**

**Problem 1:** Let $\lambda = (1^2, 2^3, 4), \mu = (3, 4, 5) \vdash 12$. Find all of the elements of $B_{\lambda, \mu}$.

**Problem 2:** Let $S(n, k)$ denote the $n, k^{th}$ Stirling number of the second kind. Use the Involution Principle to show that for $1 \leq k \leq n$,

$$k!S(n, k) = \sum_{i=0}^{k} (-1)^i \binom{k}{i} (k - i)^n.$$ 

**Problem 3:** Suppose that $\gamma : \Lambda \rightarrow \mathbb{Q}[x, y]$ is a ring homomorphism such that for $n \in \mathbb{N}$,

$$\gamma(e_n) = \frac{(-1)^{n+1} y(x - y)^{n-1}}{n!}.$$ 

Show that

$$\gamma(n! h_n) = \sum_{\lambda \vdash n} \binom{n}{\lambda} |B_{\lambda,(n)}| y^{\ell(\lambda)} (x - y)^{n - \ell(\lambda)},$$

where if $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_k) \vdash n$, $\binom{n}{\lambda} = \binom{n}{\lambda_1, \lambda_2, \ldots, \lambda_k}.$

**Problem 4:** Let $n \in \mathbb{N}$ and suppose $\lambda, \mu \vdash n$. Prove that

$$\sum_{\alpha \vdash n} (-1)^{n - \ell(\alpha)} |B_{\lambda, \alpha}| (-1)^{n - \ell(\alpha)} |B_{\alpha, \mu}| = \chi(\lambda = \mu).$$