

The Big Theorem in Linear Algebra

Suppose we are given a system of n linear equations in n variables, i.e., a square system, and let $\mathbf{Ax} = \mathbf{b}$ denote the corresponding matrix equation. Then the following are equivalent.

- (1) $\mathbf{Ax} = \mathbf{b}$ has exactly one solution.
- (2) $\mathbf{Ax} = \mathbf{0}$ has only the trivial solution.
- (3) \mathbf{A} can be row reduced to the identity matrix.
- (4) $\det(\mathbf{A}) \neq 0$.
- (5) The columns of \mathbf{A} are linearly independent.
- (6) The rows of \mathbf{A} are linearly independent.
- (7) The dimension of the column space of \mathbf{A} is n .
- (8) The dimension of the row space of \mathbf{A} is n .
- (9) The eigenvalues of A are nonzero.