MATH 3326-01 **Intro to Abstract Mathematics** **Spring 2014**

Problems on Mathematical Induction

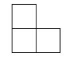
In Class: September 22 & 24

Instructions: Prove each of the following statements. When your group agrees that a proof is correct, call the instructor over to show off your masterful work.

Problem 1. Prove that for $n \in \mathbb{N}$ with $n \geq 4$, $2^n \geq n^2$.

Problem 2. Prove that every $n \in \mathbb{N}$ with $n \geq 2$ is divisible by a prime number.

Problem 3. For any $n \in \mathbb{N}$, show that $\displaystyle\sum_{i=1}^{2n} \frac{(-1)^{i+1}}{i} = \sum_{i=n+1}^{2n} \frac{1}{i}$.

Problem 4. Let $n \in \mathbb{N}$. Prove that if a single square is removed from a $2^n \times 2^n$ grid then the resulting figure can be covered by non-overlapping tiles of the form .

Problem 5. The *Fibonacci numbers* are defined by the following recurrence relation:

$$f_0 = f_1 = 1, \text{ and for } n \geq 2, \; f_n = f_{n-1} + f_{n-2}.$$

i. Prove that $f_0 + f_1 + f_2 + \cdots + f_n = f_{n+2} - 1$.

ii. Prove that $f_0 + f_2 + f_4 + \cdots + f_{2n} = f_{2n+1}$.

Problem 6. What amounts of money can be formed using only \$2 and \$5 bills? Be sure to prove that your answer is correct.

Problem 7. Given $0 \leq x \leq \pi$, show that for all $n \in \mathbb{N}$, $|\sin(nx)| \leq n\sin(x)$. You may use any of the facts below to aid in proving this fact.

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1. For $a \in \mathbb{R}$, $|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a < 0$.
2. *(The Triangle Inequality)* For $a, b \in \mathbb{R}$, $|a + b| \leq |a| + |b|$.
3. For $a, b \in \mathbb{R}$, $\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$.

Problem 8. Prove that the Principle of Mathematical Induction (Strong or otherwise) implies the Well-Ordering Principle.