Problem 38. Let \( \sim \) be an equivalence relation on a set \( A \) and suppose \( x, y \in A \). Show that if \( [x] \neq [y] \), then \( [x] \cap [y] = \emptyset \).

Problem 39. Let \( G = \{((a, b), (c, d)) \mid ad - bc = 0\} \subseteq (\mathbb{Z} \times \mathbb{N})^2 \).

i. Using only properties of the integers, i.e., without using fractions or division, show that \( G \) is an equivalence relation on \( (\mathbb{Z} \times \mathbb{N}) \).

ii. Describe \([2, 3])\).

(Hint: You may use the fact that for real numbers \( a \) and \( b \), if \( ab = 0 \), then at least one of \( a \) or \( b \) is 0.)

Problem 40. Given \( z \in \mathbb{R} \), we define the floor of \( z \), denoted by \( \lfloor z \rfloor \), to be the greatest integer less than or equal to \( z \), i.e., \( \lfloor z \rfloor = m \) if and only if \( m \leq z < m + 1 \).

i. Compute \( \lfloor \pi \rfloor \), \( \lfloor 17 \rfloor \), and \( \lfloor -103.75 \rfloor \).

ii. In class we showed that \( \sim = \{(x, y) \mid x - y \in \mathbb{Z}\} \) is an equivalence relation on \( \mathbb{R} \). Show that for any \( z \in \mathbb{R} \), \( z \sim (z - \lfloor z \rfloor) \).

iii. Show that \( \mathbb{R}/\sim = \{[y] \mid y \in [0, 1)\} \). (Note: In class we talked about how all integers are equivalent under this relation, so in particular, \([0] = [1]\). What this says, in some sense which can be made more precise in later courses, is that \( \mathbb{R}/\sim \) is a circle.)