Problem 46. Let $f : A \to B$ and $g : B \to C$ be functions and assume $U, V \subseteq A$ and $X, Y \subseteq B$. Further, define $f(U) = \{f(u) \mid u \in U\}$ and $f^{-1}(X) = \{a \in A \mid f(a) \in X\}$. Show that $f(U \cap V) \neq f(U) \cap f(V)$, but $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$.

Problem 47. Let $f : X \to Y$ and $g : Y \to Z$ be functions.

i. Suppose $g \circ f$ is injective. Prove or disprove that $f$ must be injective. Prove or disprove that $g$ must be injective.

ii. Suppose $g \circ f$ is surjective. Prove or disprove that $f$ must be surjective. Prove or disprove that $g$ must be surjective.