Problem 65. Show that every cyclic group is Abelian.

Problem 66. Let $G$ be a group and suppose $x \in G$. The order of $x$ is the smallest $n \in \mathbb{N}$ such that $x^n = e$, denoted by $|x| = n$, and if no such $n$ exists, then we say that the order of $x$ is infinite, denoted by $|x| = \infty$. For each of the following groups, compute the order of the given element $x$.

i. $(\mathbb{R}^*, \cdot), x = 1$

ii. $(\mathbb{R}, +), x = 1$

iii. $(\mathbb{Z}_{18}, +_{18}), x = 2$

iv. $(\mathbb{Z}_7, +_7), x = 2$

v. $(D_4, \circ), x = R_{180} = \text{a counterclockwise rotation of } 180^\circ$

Problem 67. For iii. and iv. in the previous problem, compute $\langle 2 \rangle$. Use your result to make (but not prove) a conjecture about the size of $\langle x \rangle$ relative to $|x|$.