Problem 53:

Let \( f : X \to Y \) be a function and suppose that the functions \( g : Y \to X \) and \( h : Y \to X \) are both inverses for \( f \), i.e. that \( f \circ g = f \circ h = I_Y \) and \( g \circ f = h \circ f = I_X \). Prove that \( g = h \).

Problem 54:

Let \( f : X \to Y \) and \( g : Y \to Z \) be functions.

a. Prove that if \( f \) and \( g \) are both injective then so is \( g \circ f \).

b. Prove that if \( f \) and \( g \) are both surjective then so is \( g \circ f \).

c. If \( g \circ f \) is injective do either of \( f \) or \( g \) have to be injective? Prove your answer.

d. If \( g \circ f \) is surjective do either of \( f \) or \( g \) have to be surjective? Prove your answer.

Problem 55:

Prove that the function \( f : \mathbb{R} \setminus \{2\} \to \mathbb{R} \) defined by \( f(x) = x/(x-2) \) is not a bijection. Find a set \( Y \subset \mathbb{R} \) so that the function \( \hat{f} : \mathbb{R} \setminus \{2\} \to Y \) given by the same formula is a bijection, and find \( \hat{f}^{-1} \).