Problem 31:

Let $A, B, C$ be sets. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Problem 32:

Let $A_1, A_2, \ldots, A_n$ be subsets of a universal set $X$. Prove DeMorgan’s Laws:

a. $\left( \bigcup_{i=1}^{n} A_i \right)^c = \bigcap_{i=1}^{n} A_i^c$.

b. $\left( \bigcap_{i=1}^{n} A_i \right)^c = \bigcup_{i=1}^{n} A_i^c$.

Problem 33:

Let $A$ and $B$ be sets.

a. Use Venn diagrams to conjecture a relationship between $(A \cup B) - (A \cap B)$ and $(A - B) \cup (B - A)$.

b. Prove your conjecture.

Problem 34:

a. Is the set difference associative? That is, is it true that $A - (B - C) = (A - B) - C$ for all sets $A, B, C$? Prove your assertion.

b. The symmetric difference of two sets $A$ and $B$ is defined to be $A \Delta B = (A - B) \cup (B - A)$. Is the symmetric difference associative? That is, is it true that $A \Delta (B \Delta C) = (A \Delta B) \Delta C$ for all sets $A, B$ and $C$? Prove your assertion.