Problem 7. Let $m, n \in \mathbb{N}$, and suppose $X$ is a set with exactly $m$ elements and $Y$ is a set with exactly $n$ elements. If $f : X \to Y$ is a bijection, which of the following may be true?

a. $n > m$.

b. $n = m$.

c. $n < m$.

d. There’s no way to tell.

Problem 8. Let $f : X \to Y$ and $g : Y \to Z$ be functions.

a. Prove that if $f$ and $g$ are both surjective then so is $g \circ f$.

b. If $g \circ f$ is injective, do either of $f$ or $g$ have to be injective? Prove your answer.

c. If $g \circ f$ is surjective, do either of $f$ or $g$ have to be surjective? Prove your answer.

Problem 9. Let $n \in \mathbb{N}$ and suppose $i, j \in I_n$. Define $h : I_n \to I_n$ by

$$h(x) = \begin{cases} 
x & \text{if } x \neq i \text{ and } x \neq j, \\
j & \text{if } x = i, \text{ and} \\
i & \text{if } x = j.
\end{cases}$$

Prove that $h$ is a bijection.