Problem 24. Suppose that $X \subseteq \mathbb{Z}$ with $X$ nonempty. Show that if $X$ is bounded below, then $X$ has a least element.

*Hint: Since $X$ is bounded below, there exists $M \in \mathbb{Z}$ such that $x \geq M$ for every $x \in X$. Now consider the function $f : X \rightarrow \mathbb{N}$ by $x \mapsto x - M + 1$. (You may begin the proof with these two sentences, if you like.) Now use the WOP to show that $f(X) = \{ f(x) \mid x \in X \}$ has a least element. Also notice that $f$ is injective. These two facts should make it so that you can find the least element of $X$."

Problem 25. Define $B(S)$ to denote the set of all bijections from $s$ to itself. Show that $L = \{ g \in B(\mathbb{R}) \mid g(x) = ax + b \text{ with } a \in \mathbb{Q}^*, b \in \mathbb{Q} \}$ is a subgroup of $(B(\mathbb{R}), \circ)$. Show that if $\mathbb{Q}^*$ is replaced with $\mathbb{Z}$, then this is not a group. (*Hint: Be aware of the operation here when you talk about “xy”.*)

Problem 26. Suppose that $H$ and $K$ are subgroups of $G$. Show that $H \cap K$ is a subgroup of $G$.\"