Consider the binary representations of real numbers, and in particular, those in the interval $(0, 1)$. That is, for $r \in (0, 1)$, we may write
\[ r = \sum_{i=1}^{\infty} \frac{a_i}{2^i}, \]
where $a_i = 0$ or $1$ for every $i \in \mathbb{N}$. For example, $.5 = \frac{1}{2} = 0.1000\ldots$, and in this case, $a_1 = 1$ and $a_j = 0$ for every $j \geq 2$. However, from Calculus II we know that we can also write
\[ .5 = \frac{1}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = \sum_{i=2}^{\infty} \frac{1}{2^i} = 0.0\overline{11} \ldots, \]
that is, $a_1 = 0$ and $a_j = 1$ for every $j \geq 2$. So, $.5$ has two binary decimal representations, one which terminates and one which is infinite.

**Problem 11.** (a) Find two binary expansions of the real number $x = 0.34375$.

(b) Show that in a binary expansions $0.01$ and $0.00\overline{11}$ represent the same real number.

(c) Using the fact that $\frac{x}{1-x} = \sum_{i=1}^{\infty} x^i$ for every $|x| < 1$, show that the binary representation of $\frac{1}{7}$ is $0.00100\overline{1}$. (Hint: $7 = 2^3 - 1$.)

**Problem 12.** (a) Use Cantor’s diagonalization method to show that the set of infinite sequences of 0’s and 1’s is uncountable. That is, $\mathcal{B} = \{x_1x_2x_3\ldots \mid x_i = 0 \text{ or } 1 \text{ } \forall i \in \mathbb{N}\}$ is uncountable.

(b) Notice that if we remove the sequences $000\ldots$ and $111\ldots$ from $\mathcal{B}$ then each of the remaining sequences corresponds to a binary decimal representation of some $x \in (0, 1)$. For example, the sequence $001\overline{001} \in \mathcal{B}$ corresponds to the binary decimal $0.001\overline{001} = \frac{1}{7}$. So, does your proof in part (a) show that $(0, 1)$ is uncountable? Explain your response.