Problem 15. On the last assignment you found a characterization of *infimum* as follows:

**Theorem:** Let $A$ be a nonempty subset of $\mathbb{R}$ which is bounded below and let $s \in \mathbb{R}$. Then $s = \inf A$ if and only if $s$ is a lower bound of $A$ and for every $\varepsilon > 0$, there is $a \in A$ such that $s + \varepsilon > a$.

Prove this theorem.

Problem 16. Determine, with justification, each of the following:

(a) $\inf \mathbb{N}$

(b) $\sup \{2 - \frac{1}{n} \mid n \in \mathbb{N}\}$

(c) $\inf \mathbb{R}^+$

Problem 17. Prove the following corollary to the Archimedean Property: *If a is a positive real number, then there exists $n \in \mathbb{N}$ such that $0 < \frac{1}{n} < a$. 