Planet Problem Solution

Solution: Let $P(n)$ be the statement that if $2n - 1$ planets are in a planetary system, each with a single astronomer observing the nearest planet, then at least one planet is unobserved. Now, $P(1)$ is true since if there is only one planet in a system, then that planet must be unobserved. So assume that $P(k)$ is true for some $k \in \mathbb{N}$, that is, if $2k + 1$ planets are in a planetary system, each with a single astronomer observing the nearest planet, then at least one planet is unobserved.

Assume there are $2(k + 1) + 1 = 2k + 3$ planets in some system, and find the two planets with the smallest distance between them (which we can do since all the planets have different distances between them). Call these planets $A$ and $B$, and notice that the astronomer on $A$ must observe $B$ and the astronomer on $B$ must observe $A$. Now, call the other $2k + 1$ planets in the system $p_1, p_2, \ldots, p_{2k+1}$. Now planets $p_1, \ldots, p_{2k+1}$ form a planetary system with $2k + 1$ planets and so at least one of those planets is unobserved by others in the system, call this planet $C$. When we add back planets $A$ and $B$, planet $C$ will remain unobserved, and so $P(k + 1)$ is true.

Thus, $P(n)$ is true for all $n \in \mathbb{N}$. 