Final Exam Review Problems

**Review Problem 0:** Review class notes and homework assignments. You should know which canonical sets are countable/uncountable, as well as what set operations preserve countability.

**Review Problem 1:** Give definitions for the following terms: countable, uncountable, bounded above, bounded below, supremum, infimum. You should also know the working definitions of any terms, when applicable.

**Review Problem 2:** State the Axiom of Completeness for $\mathbb{R}$. Show that the Axiom of Completeness holds for $\mathbb{Z}$, but that it does not hold for $\mathbb{Q}$.

**Review Problem 3:** State the Archimedean Property.

**Review Problem 4:** Find the $\sup(A)$, where $A = \{\frac{2n+1}{n+1} \mid n \in \mathbb{N}\}$, or state why it does not exist. Do the same for $\inf(A)$.

**Review Problem 5:** Find the $\sup(A)$, where $A = \{\frac{1}{n} - \frac{1}{m} \mid m, n \in \mathbb{N}\}$, or state why it does not exist.

**Review Problem 6:** Show that the set of irrational numbers is uncountable.

**Review Problem 7:** Show by example that if $X_1, X_2, X_3, \ldots$ are countable sets, that the product of these sets, $\prod_{i=1}^{\infty} X_i = X_1 \times X_2 \times X_3 \times \cdots$, need not be.

**Review Problem 8:** For a positive integer $n$, let $B(n)$ be the set of subsets of $\mathbb{N}$ having exactly $n$ elements. Show that for all $n \geq 1$, $B(n)$ is countable.

**Review Problem 9:** Show that the union of a countable set and an uncountable set is uncountable.

**Review Problem 10:** Let $A, B$ be nonempty subsets of $\mathbb{R}$ such that for all $x \in A$ and $y \in B$, we have that $x \leq y$. Show that $\sup A \leq \inf B$. Furthermore, show that $\sup A = \inf B$ if and only if for all $\epsilon > 0$, we may find $x \in A$ and $y \in B$ such that $y - x < \epsilon$. 