Problem 39. Pick \( k \in \mathbb{N} \) and define \( M_k = \{(x, y) \mid k|(x - y)\} \subseteq \mathbb{Z}^2 \). Show that \( M_k \) is an equivalence relation on \( \mathbb{Z} \) and describe \([6]\) on the set \( M_4 \). Is this the same as the set \([6]\) on \( M_5 \)? Explain. This equivalence is called congruence modulo \( k \).

Problem 40. For \( x, y \in \mathbb{R} \), define \( \sim \) by \( x \sim y \) whenever there exists \( r \in \mathbb{R} \) with \( r \neq 0 \) such that \( x = yr^2 \). For any \( a \in \mathbb{R} \), describe \([a]\).

Problem 41. If \( A = \{1, 2, 3, \ldots, 9, 10\} \) and \( S = \{\{1, 3, 5\}, \{2, 8\}, \{4, 6, 9, 10\}, \{7\}\} \), find an equivalence relation on \( A \) whose set of equivalence classes is exactly \( S \).