Let $f : A \to B$ and $g : B \to C$ be functions and assume $U, V \subseteq A$ and $X, Y \subseteq B$. Further, define $f(U) = \{ f(u) \mid u \in U \}$ and $f^{-1}(X) = \{ a \in A \mid f(a) \in X \}$.

**Problem 51.** Prove or disprove that if $g \circ f$ and $f$ are bijective, then $g$ is bijective. Note, you are not told that any of the sets $A$, $B$, or $C$ are finite, so you cannot use Problem 48.

**Problem 52.** Show that $f(U \cap V) \neq f(U) \cap f(V)$, but $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$. 