Problem 1. (From the book.) Chapter 11 Problem Set, page 93: 9, 10, 12, 15-17, 19

Problem 2. Let $p$ be an odd prime.

i. Show that $(-2/p) = 1$ if $p \equiv 1, 3 \pmod{8}$ and $(-2/p) = -1$ if $p \equiv 5, 7 \pmod{8}$.

ii. Show that $(-3/p) = 1$ if $p \equiv 1 \pmod{6}$ and $(-3/p) = -1$ if $p \equiv 5 \pmod{6}$ whenever $p > 3$.

iii. From a previous homework we saw that if $p$ is an odd prime that divides $n^2 + 1$, then $p = 4k + 1$ for some $k \in \mathbb{N}$. Determine which integers can divide integers of the forms $n^2 + 2$ or $n^2 + 3$.

Problem 3. Let $a, b \in \mathbb{N}$ with $b$ odd so that $(a, b) = 1$, and suppose $b = p_1p_2\cdots p_k$, where the $p_i$'s are primes which are not necessarily distinct. Define the Jacobi symbol $(a \mid b)$ by

$$(a \mid b) = (a/p_1)(a/p_2)\cdots(a/p_k).$$

i. Compute $(21 \mid 221)$ and $(35 \mid 4437)$.

ii. Show that if $(a/b) = 1$, then $(a \mid b) = 1$, but that the converse of this statement is not true.